

Oppgave 1:

$$f'(x) = \lim_{h \rightarrow \infty} \frac{(3(x+h) - (x+h)^2) - (3x - x^2)}{h}$$

$$f'(x) = \lim_{h \rightarrow \infty} \frac{(3x - 3h - (x^2 + 2xh + h^2)) - 3x + x^2}{h}$$

$$f'(x) = \lim_{h \rightarrow \infty} \frac{3x + 3x - x^2 - 2xh - h^2 - 3x + x^2}{h}$$

Stryker i telleren:

$$f'(x) = \lim_{h \rightarrow \infty} \frac{3h - 2xh - h^2}{h} = \frac{h(3 - 2x - h)}{h} = 3 - 2x - h = 3 - 2x$$

$$f'(x) = 3 - 2x$$

Oppgave 2:

a)

$$f(x) = \frac{3x - x^2}{\sqrt{x}}$$

Bruker brøkregelen ($\frac{u'v - v'u}{v^2}$):

$$f'(x) = \frac{(3x - x^2)' \sqrt{x} - (3x - x^2)(\sqrt{x})'}{(\sqrt{x})^2}$$

$$f'(x) = \frac{(3 - 2x)\sqrt{x} - (3x - x^2) \frac{1}{2\sqrt{x}}}{(\sqrt{x})^2}$$

$$f'(x) = \frac{3 - 2x}{\sqrt{x}} - \frac{3x - x^2}{(\sqrt{x})^2}$$

$$f'(x) = \frac{3 - 2x}{\sqrt{x}} - \frac{3x - x^2}{2\sqrt{x}} \cdot \frac{1}{x} = \frac{3 - 2x}{\sqrt{x}} - \frac{3x - x^2}{2x\sqrt{x}}$$

$$f'(x) = \frac{(3 - 2x) \cdot 2x}{2x\sqrt{x}} - \frac{3x - x^2}{2x\sqrt{x}} = \frac{6x - 4x^2 - 3x + x^2}{2x\sqrt{x}}$$

$$f'(x) = \frac{3x - 3x^2}{2x\sqrt{x}} = \frac{3x(1 - x)}{2x\sqrt{x}} = \frac{3(1 - x)}{2\sqrt{x}}$$

b)

$$f(x) = \frac{3x - x^2}{\sqrt{x}} = \frac{3x}{\sqrt{x}} - \frac{x^2}{\sqrt{x}} = 3x^{(1-0.5)} - x^{(2-0.5)} = 3\sqrt{x} - x^{(3/2)}$$

$$(3\sqrt{x} - x^{(3/2)})' = 3 \frac{1}{2\sqrt{x}} - \frac{3}{2} x^{(0.5)}$$

$$= \frac{3}{2\sqrt{x}} - \frac{3\sqrt{x}}{2}$$

$$= \frac{3}{2\sqrt{x}} - \frac{3\sqrt{x} \cdot \sqrt{x}}{2 \cdot \sqrt{x}} = \frac{3}{2\sqrt{x}} - \frac{3x}{2\sqrt{x}} = \frac{3(1 - x)}{2\sqrt{x}}$$

Oppgave 3:

$$h(x) = \cos^2 x + \sin^2 x$$

 $\cos^2 x + \sin^2 x$ er definert som 1, derfor er

$$h(x) = 1$$

$$h'(x) = 0$$