# TDT4136 Logic and Reasoning Systems

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Assignment 2

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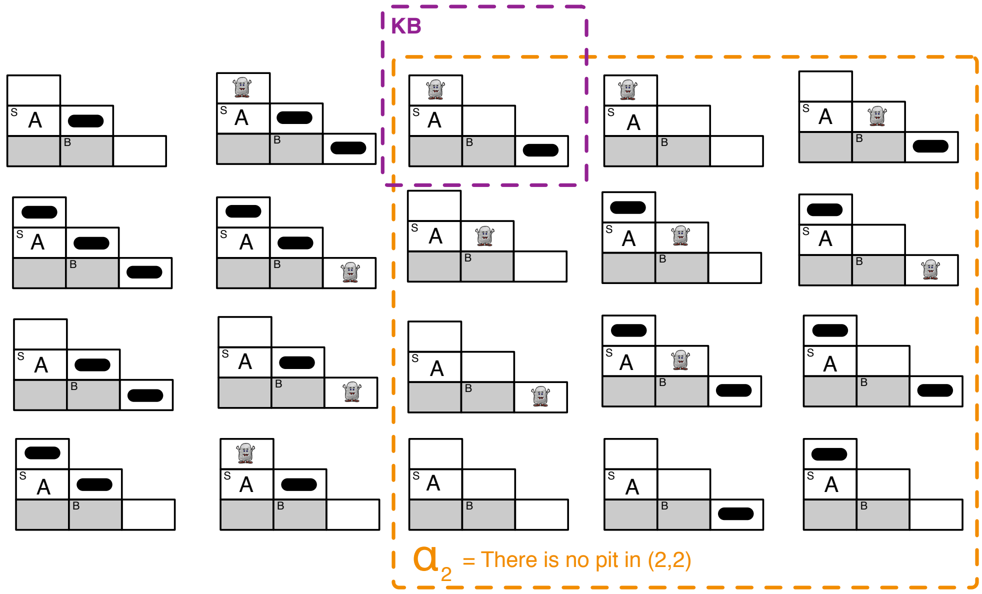
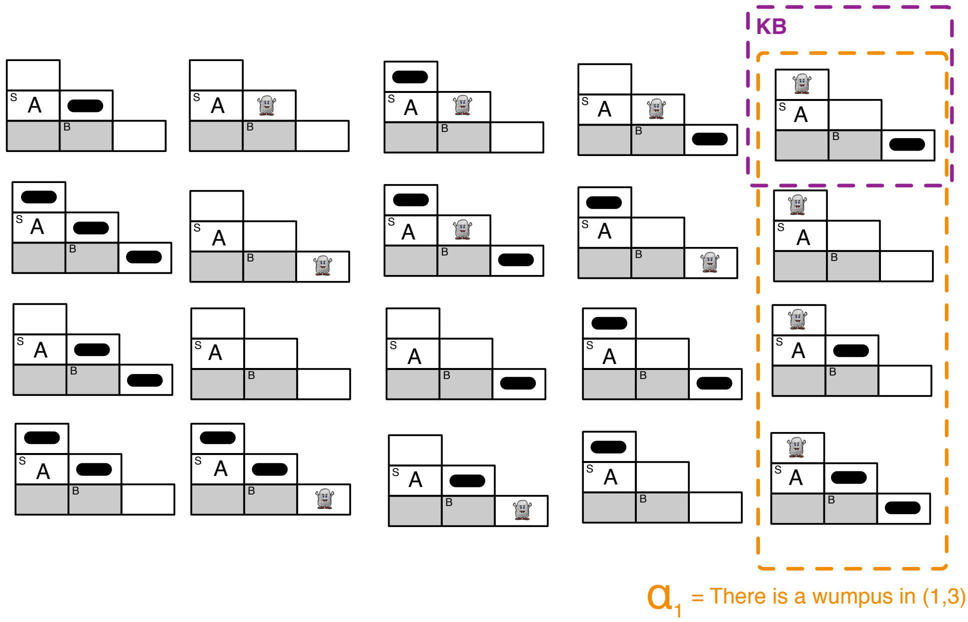
# 1 Page 279, Exercise 7.1



## 2 Page 280, Exercise 7.4

*I drafted the truth tables by hand.*

**A)** $False \models True$ **correct?**

Since the truth set of False is a subset of the truth set of True, I believe this is correct.

**B)** $True \models False$ **incorrect**

**C)** $(A \wedge B) \models (A \iff B)$ **correct**

The values on both side of the "equal sign" turns the expression into a *True entails True* or *False entails False* - which is correct.

**D)** $A \iff B \models A \vee B$ **incorrect**

This fails both A and B are false, which would make $A \iff B$ True and $A \vee B$ False.

**E)** $A \iff B \models \neg A \vee B$ **correct**

This entailment would be incorrect if rearranged. $A \iff B$ has a subset of the "truth set" of $\neg A \vee B$.

**F)** $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B \vee C) \wedge (B \wedge C \wedge (D \rightarrow E))$ **incorrect.**

The left hand side of the entailment contains combinations which yields True, while the right hand side would yield False. This contradict *entailment*. Eg. the combination $A \neg B \neg C D \neg E$

**G)** $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B) \wedge (\neg D \vee E)$ **correct.**

The two truth tables are equivalent.

**H)** $(A \vee B) \wedge \neg(A \rightarrow B)$ **is satisfiable**

This expression is satisfiable for $A \neg B$

**I)** $(A \vee B) \rightarrow C \models (A \rightarrow C) \vee (B \rightarrow C)$ **correct.**

Checked by using truth tables.

**J)** $(C \vee (\neg A \wedge \neg B)) \equiv ((A \rightarrow C) \wedge (B \rightarrow C))$ **incorrect.**

The two truth tables differ on the combinations $(A \neg B \neg C)$ and $(\neg A B \neg C)$.

**K)** $(A \iff B) \wedge (\neg A \vee B)$ **is satisfiable.**

This is satisfiable for $AB$ and $\neg A \neg B$

**L)**

*I did not understand the formulation in this exercise.*

# 3 Page 281, Exercise 7.7

**A)** $B \lor C$

$2^2$ models

**B)** $\neg A \lor \neg B \lor \neg C \lor \neg D$

$2^4$ models

**C)** $(A \to B) \land A \land \neg B \land C \land D$

$2^4$ models

# 4 Page 281, Exercise 7.10

**A)** $A \to A \equiv A \lor \neg A$ **valid**

**B)** $A \to B$ **satisfiable**

**C)** $(A \to B) \to (\neg A \to \neg B)$ **satisfiable**

**D)** $A \lor B \lor \neg B$ **valid**

**E)** $((A \land B) \to C) \leftrightarrow ((A \to C) \lor (B \to C))$ **valid**

**F)** $A \lor B \lor (A \to B)$ **valid**

**G)** $(A \land B) \lor \neg B$ **satisfiable**

# 5 Exercise

**Consider a logical knowledge base with 100 variables, $A_1, A_2, ...A_{100}$. This will have $Q = 2^{100}$ possible models. For each logical sentence below, give the number of models that satisfy it. Feel free to express you answer as a fraction of $Q$ (without writing out the whole number $1267650600228229401496703205376 = 2^{100}$) or to use other symbols to represent large numbers.**

**A)** $A_1 \vee A_{73}$ **valid models:** $\frac{3}{4}Q$

**B)** $A_7 \vee (A_{19} \wedge A_{33})$ **valid models:** $\frac{5}{8}Q$

**C)** $A_{11} \rightarrow A_{22} \equiv \neg A_{11} \vee A_{22}$ **this is in essence the same as A):** $\frac{3}{4}Q$

# 6 Convert each of the following sentences to Conjunctive Normal Form (CNF).

**A)** $A \wedge B \wedge C$ **CNF:** $A \wedge B \wedge C$

**B)** $A \vee B \vee C$ **CNF:** $A \vee B \vee C$

**C)** $A \rightarrow (B \vee C)$ **CNF:** $\neg A \vee B \vee C$

**D)** $(A \vee \neg C) \rightarrow B$ **CNF:** $(\neg A \vee B) \wedge (C \vee B)$

**G)** $(A \wedge B) \leftrightarrow C$ **CNF:** $(\neg A \vee \neg B \vee C) \wedge (A \vee \neg C) \wedge (B \vee \neg C)$

# 7 Consider the following Knowledge Base (KB) and use resolution to show that $KB \models \neg E$:

- $(A \vee \neg B) \rightarrow \neg C$
- $(D \wedge E) \rightarrow C$
- $A \wedge D$

**We start off by converting the KB to CNF:**

**I** $(\neg A \vee \neg C) \wedge (B \vee \neg C)$

**II** $\neg D \vee \neg E \vee C$

**III** $A \wedge D$

**Then we have to negate our conclusion since we are proving by contradiction.**

**IV** $E$

**Resolution:**

**0** $\neg E$

**1** $A = True$ **from III**

**2** $D = True$ **from III, 1**

**3** $C = True$ **from II, 2, IV**

**4** **I is consistent False as a consequence of our bounded variables.**

**E can't be false, proven by contradiction.**

## 8 Page 316, Exercise 8.9

**A) Paris and Marseilles are both in France.**

- **i** $In(Paris \land Marseilles, France)$ **This is not a valid syntax.**
- **ii** $In(Paris, France) \land In(Marseilles, France)$ **Correctly expresses the English sentence.**
- **iii** $In(Paris, France) \lor In(Marseilles, France)$ **Is syntactically valid, but doesn't express the sentence.**

**B) There is a country that borders both Iraq and Pakistan.**

- **i** $\exists c(Country(c) \land Border(c, Iraq) \land Border(c, Pakistan))$ **Correctly expresses the English sentence.**
- **ii** $\exists c(Country(c) \Rightarrow [Border(c, Iraq) \land Border(c, Pakistan)])$ **This sentence says: there is a country c that borders Iraq and Pakistan. But also says: if c isn't a country, it borders Iraq and Pakistan.**
- **iii** $\exists c(Country(c)) \Rightarrow [Border(c, Iraq) \land Border(c, Pakistan)]$ **This is not a valid syntax.**
- **iv** $\exists c(Border(Country(c, Iraq \land Pakistan)))$ **This is not a valid syntax.**

**C) All countries that border Ecuador are in South America.**

- **i** $\forall c(Country(c) \land Border(c, Ecuador) \Rightarrow In(c, SouthAmerica))$ **Correctly expresses the English sentence.**
- **ii** $\forall c(Country(c) \Rightarrow [Border(c, Ecuador) \Rightarrow In(c, SouthAmerica)]$ **Correctly expresses the English sentence.**
- **iii** $\forall c([Country(c) \Rightarrow Border(c, Ecuador)]) \Rightarrow In(c, SouthAmerica))$ **Is syntactically valid, but doesn't express the sentence.**
- **iv** $\forall c(Country(c) \land Border(c, Ecuador) \land In(c, SouthAmerica))$ **Is syntactically valid, but doesn't express the sentence.**

**D) No region in South America borders any region in Europe.**

- **i** $\neg[\exists c, d\ In(c, SouthAmerica) \land In(d, Europe) \land Borders(c, d)]$ **Correctly expresses the English sentence.**
- **ii** $\forall c, d\ [In(c, SouthAmerica) \land In(d, Europe)] \Rightarrow \neg Borders(c, d)]$ **Correctly expresses the English sentence.**
- **iii** $\neg \forall c In(c, SouthAmerica) \Rightarrow \exists d\ In(d, Europe) \land \neg Borders(c, d)$ **Is syntactically valid, but doesn't express the sentence.**
- **iv** $\forall c\ In(c, SouthAmerica) \Rightarrow \forall d\ In(d, Europe) \Rightarrow \neg Borders(c, d)$. **This is not a valid syntax.**

## 9 Page 319, Exercise 8.21

**In Chapter 6, we used equality to indicate the relation between a variable and its value. For instance, we wrote $WA = red$ to mean that Western Australia is colored red. Representing this in first-order logic, we must write more verbosely $ColorOf(WA) = red$ . What incorrect inference could be drawn if we wrote sentences such as $WA = red$ directly as logical assertions?**

## 10 Page 319, Exercise 8.23 - Feel free to use several logical sentences to express this one natural-language statement.

**For each of the following sentences in English, decide if the accompanying first-order logic sentence is a good translation. If not, explain why not and correct it. (Some sentences may have more than one error!)**

**i No two people have the same social security number.**
$\neg\exists x, y, n \ \ Person(x) \wedge Person(y) \ \Rightarrow \ [HasSS\#(x, n) \wedge HasSS\#(y, n)]$
**This is incorrect. The correct sentence is:**
$\neg\exists x, y, n \ Person(x)\wedge Person(y)\wedge\neg(x = y)\wedge HasSS\#(x, n)\wedge HasSS\#(y, n)$

**ii John's social security number is the same as Mary's.**
$\exists n \ HasSS\#(John, n) \wedge HasSS\#(Mary, n)$
**This logic sentence is spotless.**

**iii Everyone's social security number has nine digits.**
$\forall x, n \ Person(x) \Rightarrow [HasSS\#(x, n) \wedge Digits(n, 9)]$
**This sentence says that every person has a SS# which contains 9 digits. By moving the $HasSS\#()$ predicate we achieve the intended meaning:**
$\forall x, n \ (Person(x) \wedge HasSS\#(x, n)) \Rightarrow \wedge Digits(n, 9)$

**iv Rewrite each of the above (uncorrected) sentences using a function symbol $SS\#$ instead of the predicate $HasSS\#$**

- **i** $\neg\exists x, y \ Person(x) \wedge Person(y) \Rightarrow SS\#(x) = SS\#(y)$
- **ii** $SS\#(John) = SS\#(Mary)$
- **iii** $\forall x \ Person(x) \Rightarrow Digits(SS\#(x), 9)$

## 11 Page 319, Exercise 8.24

**Represent the following sentences in first-order logic, using a consistent vocabulary (which you must define):**

**A) Some students took French in spring 2001.**
$\exists s \ Student(s) \wedge HasSubject(s, French, 2001SPRING)$

**B) Every student who takes French passes it.**
$\forall s, semester\ Student(s) \wedge HasSubject(s, French, semester) \Rightarrow Passed(s, French, semester)$

**C) Only one student took Greek in spring 2001.**
$\exists s\ \forall x\ HasSubject(s, Greek, 2001SPRING) \wedge \neg(s = x) \Rightarrow \neg HasSubject(x, Greek, 2001SPRING)$

**D) The best score in Greek is always higher than the best score in French.**
$\exists s\ \forall y, semester\ Score(s, Greek, semester) > Score(y, French, semester)$

**E) Every person who buys a policy is smart.**
$\forall person\ \exists pol\ Person(person) \wedge (Policy(pol) \wedge Buy(person, pol)) \Rightarrow Smart(person)$

**F) No person buys an expensive policy.** $\forall person, pol\ Person(person) \wedge Policy(pol) \wedge Expensive(pol) \Rightarrow \neg Buy(person, pol)$

**G) There is an agent who sells policies only to people who are not insured.**
$\exists agent\ \forall person, pol\ Agent(agent) \wedge Policy(pol) \wedge Sell(agent, pol, person) \Rightarrow Person(person) \wedge \neg Insured(person)$

**H) There is a barber who shaves all men in town who do not shave themselves.**
$\exists barber\ \ \forall man\ \ Man(man) \wedge \neg Shave(man, man) \wedge Barber(barber) \Rightarrow Shave(barber, man)$

**I) A person born in the UK, each of whose parents is a UK citizen or a UK resident, is a UK citizen by birth.**
$\forall person\ Person(person) \wedge Born(person, UK) \wedge (\forall parent\ Parent(parent, person) \Rightarrow (\exists reason\ Citizen(parent, UK, reason)) \vee Resident(parent, UK)) \Rightarrow Citizen(person, UK, Birth)$

**J) A person born outside the UK, one of whose parents is a UK citizen by birth, is a UK citizen by descent.**
$\exists person\ Person(person) \wedge \neg Born(person, UK) \wedge (\exists parent, reason\ Parent(parent, person) \wedge Citizen(parent, UK, reason)) \Rightarrow Citizen(person, UK, Descent)$

**K) Politicians can fool some of the people all of the time, and they can fool all of the people some of the time, but they can't fool all of the people all of the time.**
$\forall politician \Rightarrow [\exists person\ \forall time\ Person(person) \wedge Fool(politician, person, time)] \wedge [\exists time\ \forall person\ Person(person) \Rightarrow Fool(politician, person, time)] \wedge \neg(\forall person, timePerson(person) \Rightarrow Fool(politician, person, time))$

**L) All Greeks speak the same language. (Use Speaks(x,l) to mean that person x speaks language l.)**
$\exists language\ (\forall person\ [\exists reason\ Citizen(person, Greece, reason)] \wedge Resident(person, Greece) \Rightarrow Speak(person, language))$

## 12 Find the unifier ($\Theta$) - if possible - for each pair of atomic sentences. Here, Owner, Horse and Rides are predicates, while FastestHorse is a function that maps a person to the name of their fastest horse:

**A)** $Horse(x)...Horse(Rocky)$
$\Theta(x/Rocky)$

**B)** $Owner(Leo, Rocky)...Owner(x, y)$
$\Theta(x/Leo, y/Rocky)$

**C)** $Owner(Leo, x)...Owner(y, Rocky)$
$\Theta(y/Leo, x/Rocky)$

**D)** $Owner(Leo, x)...Rides(Leo, Rocky)$
$\Theta()$

**E)** $Owner(x, FastestHorse(x))...Owner(Leo, Rocky)$
$\Theta(x/Leo)$

**F)** $Owner(Leo, y)...Owner(x, FastestHorse(x))$
$\Theta(x/Leo, y/FastestHorse(Leo))$

**G)** $Rides(Leo, FastestHorse(x))...Rides(y, FastestHorse(Marvin))$
$\Theta(x/Marvin, y/Leo)$

## 13 Use resolution to prove Green(Linn) given the information below. You must first convert each sentence into CNF. Feel free to show only the applications of the resolution rule that lead to the desired conclusion. For each application of the resolution rule, show the unification bindings, $\Theta$.

**I** $Hybrid(Prius)$

**II** $Drives(Linn, Prius)$

**III** $\forall x\ Green(x) \Leftrightarrow Bikes(x) \vee [\exists y\ Drives(x, y) \wedge Hybrid(y)]$

**We start off by converting the sentences into CNF.**

**i** $Hybrid(Prius) \xrightarrow{CNF} Hybrid(Prius)$

**ii** $Drives(Linn, Prius) \xrightarrow{CNF} Drives(Linn, Prius)$

**iii** $\forall x\ Green(x) \Leftrightarrow Bikes(x) \vee [\exists y\ Drives(x,y) \wedge Hybrid(y)] \xrightarrow{CNF} [\neg Green(x) \vee Bikes(x) \vee Drives(x, C(x))] \wedge [\neg Green(x) \vee Bikes(x) \vee Hybrid(C(x))] \wedge [\neg Bikes(x) \vee Green(x)] \wedge [\neg Drives(x,z) \vee \neg Hybrid(z)) \vee Green(x)]$

**Removing the equivalence and introducing implications.**
$\forall x\ (Green(x) \Rightarrow Bikes(x) \vee [\exists y\ Drives(x,y) \wedge Hybrid(y)]) \wedge (Bikes(x) \vee [\exists y\ Drives(x,y) \wedge Hybrid(y)] \Rightarrow Green(x))$

**Removing the implications**
$\forall x\ \neg Green(x) \vee [Bikes(x) \vee (\exists y\ Drives(x,y) \wedge Hybrid(y))] \wedge [\neg Bikes(x) \wedge (\forall y\ \neg Drives(x,y) \vee \neg Hybrid(y))] \vee Green(x)$

**Standardizing variables apart**
$\forall x\ \neg Green(x) \vee [Bikes(x) \vee (\exists y\ Drives(x,y) \wedge Hybrid(y))] \wedge [\neg Bikes(x) \wedge (\forall z\ \neg Drives(x,z) \vee \neg Hybrid(z))] \vee Green(x)$

**Skolemization:**
C(x) is a *Skolem function*

**(Am I supposed to introduce the variable z?)**

$\forall x\ \neg Green(x) \vee [Bikes(x) \vee (Drives(x, C(x)) \wedge Hybrid(C(x)))] \wedge [\neg Bikes(x) \wedge (\forall z\ \neg Drives(x,z) \vee \neg Hybrid(z))] \vee Green(x)$

**Move the quantifiers outwards:**
$\forall x\ \forall z\ \neg Green(x) \vee [Bikes(x) \vee (Drives(x, C(x)) \wedge Hybrid(C(x)))] \wedge [\neg Bikes(x) \wedge (\neg Drives(x,z) \vee \neg Hybrid(z))] \vee Green(x)$

**Drop all universal quantifiers:**
$\neg Green(x) \vee [Bikes(x) \vee (Drives(x, C(x)) \wedge Hybrid(C(x)))] \wedge [\neg Bikes(x) \wedge (\neg Drives(x,z) \vee \neg Hybrid(z))] \vee Green(x)$

**Distribute $\wedge$ and $\vee$:**
$[\neg Green(x) \vee Bikes(x) \vee Drives(x, C(x))] \wedge [\neg Green(x) \vee Bikes(x) \vee Hybrid(C(x))] \wedge [\neg Bikes(x) \vee Green(x)] \wedge [\neg Drives(x,z) \vee \neg Hybrid(z)) \vee Green(x)]$

## Then negate the sentence we are trying to prove.

$Green(Linn) \xrightarrow{negate} \neg Green(Linn)$

**iv** $\neg Green(Linn)$

## Resolution:

**1 Combining iv and iii:** $\Theta(x/Linn)$
$[\neg Green(Linn) \vee Bikes(Linn) \vee Drives(Linn, C(Linn))] \wedge [\neg Green(Linn) \vee Bikes(Linn) \vee Hybrid(C(Linn))] \wedge [\neg Bikes(Linn) \vee Green(Linn)] \wedge [\neg Drives(Linn, z) \vee \neg Hybrid(z)) \vee Green(Linn)]$

**2 Combining 1 and ii:** $\Theta(x/Linn, z/Prius)$
$[\neg Green(Linn) \vee Bikes(Linn) \vee Drives(Linn, C(Linn))] \wedge [\neg Green(Linn) \vee Bikes(Linn) \vee Hybrid(C(Linn))] \wedge [\neg Bikes(Linn)] \wedge [\neg Drives(Linn, Prius) \vee \neg Hybrid(Prius)]$

**3 Combining 2 and i:** $\Theta(x/Linn, z/Prius, C(Linn)/Prius)$
$[\neg Green(Linn) \vee Bikes(Linn) \vee Drives(Linn, Prius)] \wedge [\neg Green(Linn) \vee Bikes(Linn) \vee Hybrid(Prius)] \wedge [\neg Bikes(Linn)] \wedge [\ ]$