

TDT4136 Logic and Reasoning Systems

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Assignment 4

Fall 2013

1 Simulated Annealing

The search node

Since Python code is pretty self-explanatory, I have appended a few lines of comment to the script instead of writing an extended explanation here. These extended comments are found in the source script.

```
class Node:
    def __init__(self, gene):
        # The node must contain a cost value
        self.gene = gene
        self.cost = self.objective_function( gene )

    def random_neighbour(self):
        # **IMPLEMENT**
        # generate neighbors
        """
        We only care about pseudo generating the neighbors,
        since we are going to pick just one of them. Thus
        we only need to perform a "small" random alteration
        to the parent.
        """
        return neighbor

    def objective_function(self, gene ):
        # **IMPLEMENT**
        # evaluate this state

        return evaluation
```

Figure 1: The python implementation of the generalized search node

Handling the temperature variations

```
def initial_temp( node ):
    """
    f.ex upper bound on (max cost - min cost)

    The example below belongs to the Egg Box exercise.
    Returns the maximum cost possible by putting an egg in each
    of the available spots.
    """
    tmp = np.ones(shape=node.gene.shape,dtype="uint8")
    return float(node.objective_function( tmp ))

def dependant_initial_t( temperature,node,neighbor ):
    """
    Increase the temperature to make this neighbor a very likely choice.
    This helps us to "correct" the initial temperature if it was initially
    too low.
    This function may not decrease the temperature.
    """
    x = temperature/(abs(node.cost-neighbor.cost)+1)*math.log1p(100/99)
    return temperature/x if 1>x>0 else temperature

def reduce_t( temperature,repetitions ):
    """
    common function for temperatur reduction
    """
    return temperature/math.log(repetitions+2,2)
```

Figure 2: How this script handles the temperature variations.

The main search function

```
def sa( start_node, goal_cost ):
    """
    d : # of repetitions pr temperature value
        This should initially be set to something like the size( neighborhood )
        but since we're only generating a single neighbor, we're forced to
        approximate.
    """
    node = start_node
    d = start_node.gene.shape[0]**2 # number of repetitions pr temperature value
    temperature = initial_temp( node )

    repetitions = 0
    while temperature > 1e-100 and node.cost > goal_cost:
        neighbor = Node( node.random_neighbor(), k ) # choose random neigh
        if neighbor.cost < node.cost:
            node = neighbor
        else:
            annealing = random.uniform(0,1)
            if annealing < math.pow(math.e,-(neighbor.cost-node.cost)/temperature):
                """
                A stochastic evaluation for checking it its time to perform the
                annealing.
                """
                node = neighbor
            repetitions += 1

        if repetitions < d:
            # set a dependant initial temperature
            # we enter this clause during the first couple of runs
            temperature = dependant_initial_t( temperature, node, neighbor )
        elif repetitions % d == 0:
            # and then we enter this clause occasionally afterwards.
            temperature = reduce_t( temperature, repetitions )
    #end while
    return node
```

Figure 3: How this script handles the temperature variations.

2 The Egg Carton Puzzle

The specialized implementation

I made a few alterations to the generalized node to be able to constraining the k-value. I am also using the NumPy function shuffle to generate a new random child node. *Shuffle()* will move around the elements in the given row or column, and thus perform a “minor” alteration to the parent gene (as required). The idea behind the *objective function()* will be explained later.

My code required to further customizing to solve the assignment.

```
class Node:
    def __init__(self, gene, k):
        # The node must contain a cost value
        self.gene = gene
        self.k = k
        self.cost = self.objective_function( gene )

    def random_neighbour(self):
        # generate neighbors
        tmp = np.copy( self.gene )
        np.random.shuffle(tmp[random.random()*tmp.shape[0],:])
        np.random.shuffle(tmp[:,random.random()*tmp.shape[0]])
        return tmp

    def objective_function(self, gene ):
        n = gene.shape[0]
        k = self.k
        horizontal = np.sum(gene, axis=1)-k
        vertical = np.sum(gene, axis=0)-k
        diag1 = np.array([
            sum(gene.diagonal(k-n+i))
            for i in xrange(2*(n-k)+1)
        ])-k
        diag2 = np.array([
            sum(np.flipr(gene).diagonal(k-n+i))
            for i in xrange(2*(n-k)+1)
        ])-k

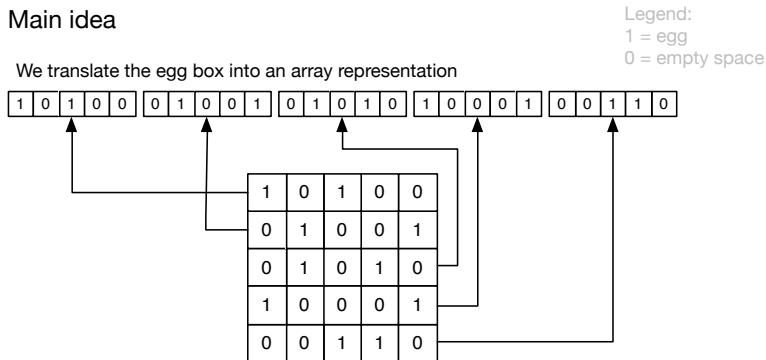
        return np.sum(horizontal[horizontal>0],axis=0) + \
            np.sum(vertical[vertical>0],axis=0) + \
            np.sum(diag2[diag2>0],axis=0) + \
            np.sum(diag1[diag1>0],axis=0)
```

Figure 4: The python implementation of the specialized search node

The problem representation

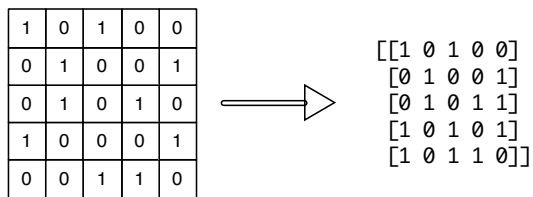
Main idea

We translate the egg box into an array representation



Actually implementation

To improve the time complexity of our search, I have implemented NumPy matrices instead



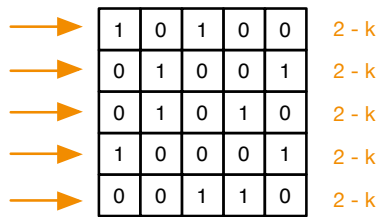
I chose this implementation strategy because it would make it a lot easier for me to perform the evaluation in the objective function. When represented like this, all we need to do is sum up the integers along a defined line.

The objective function

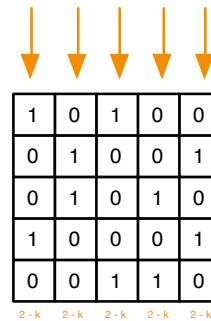
Main idea

Calculate the sum of the integers along a defined direction.

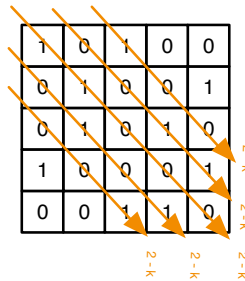
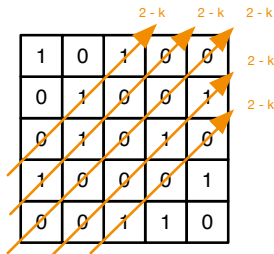
We start off by calculating the sum across all the horizontal rows, and subtract the permitted number of eggs in a single row (k)



Calculate the sum across all the vertical columns, and subtract the permitted number of eggs in a single column (k)



Calculate the sum across all the diagonals and subtract the $[k]$, starting from the diagonal with at least $[k]$ elements. For instructional purposes, I've chosen $k=3$



This function will always return a positive value, and only return 0 if we have an optimal solution. It is computationally fast, but horrific in memory if we were to encounter BIG boxes.

The execution results

Simulated Annealing - (5x5) box of 10 eggs with k=2:

```
[[0 1 0 1 0]
 [0 0 0 1 1]
 [0 1 0 0 1]
 [1 0 1 0 0]
 [1 0 1 0 0]]
```

Simulated Annealing - (6x6) box of 12 eggs with k=2:

```
[[0 0 1 1 0 0]
 [1 0 0 0 1 0]
 [0 0 1 0 1 0]
 [1 1 0 0 0 0]
 [0 0 0 1 0 1]
 [0 1 0 0 0 1]]
```

Simulated Annealing - (8x8) box of 8 eggs with k=1:

```
[[0 0 1 0 0 0 0 0]
 [0 0 0 0 1 0 0 0]
 [0 1 0 0 0 0 0 0]
 [0 0 0 0 0 0 0 1]
 [1 0 0 0 0 0 0 0]
 [0 0 0 0 0 0 1 0]
 [0 0 0 1 0 0 0 0]
 [0 0 0 0 0 1 0 0]]
```

Simulated Annealing - (10x10) box of 29 eggs with k=1:

```
[[1 0 0 0 1 0 0 1 0 0]
 [0 0 0 1 0 1 0 1 0 0]
 [0 0 1 1 0 1 0 0 0 0]
 [1 0 1 0 0 0 0 0 1 0]
 [0 0 0 0 0 1 1 0 0 1]
 [0 0 0 0 0 0 1 0 1 0]
 [1 0 0 0 1 0 0 0 0 1]
 [0 0 1 0 0 0 1 0 0 1]
 [0 1 0 0 1 0 0 0 1 0]
 [0 1 0 1 0 0 0 1 0 0]]
```

Discuss the similarities and differences between heuristics and objective functions.

A heuristic function serves as a means to describing how far this state is from the goal state. It has a definition of a goal to reach whilst a objective function looks at the given state objectively, and tells us how good this state is. The term “objective function” is used when it returns a value we want to maximize or minimize (as is done in this assignment).

The Python script

```
import random
import math
import itertools
import numpy as np

"""
minimizing

Observation:
* each node should generate the same number of neighbors
* be aware of poor performance under spiky neighborhoods
* yields better results than local search and multisstart search after t time
* should not be used on TSP or similar problems

This example will sort the digits 0123456789 #note that 0 will be removed from the integer
"""

class Node:
    def __init__(self, gene, k):
        # The node must contain a cost value
        self.gene = gene
        self.k = k
        self.cost = self.objective_function( gene )

    def random_neighbour(self):
        # generate neighbors
        tmp = np.copy( self.gene )
        np.random.shuffle(tmp[random.random()*tmp.shape[0],:])
        np.random.shuffle(tmp[:,random.random()*tmp.shape[0]])
        return tmp

    def objective_function(self, gene ):
        n = gene.shape[0]
        k = self.k
        horizontal = np.sum(gene, axis=1)-k
        vertical = np.sum(gene, axis=0)-k
        diag1 = np.array([
            sum(gene.diagonal(k-n+i))
            for i in xrange(2*(n-k)+1)
        ])-k
        diag2 = np.array([
            sum(np.fliplr(gene).diagonal(k-n+i))
            for i in xrange(2*(n-k)+1)
        ])-k

        return np.sum(horizontal[horizontal>0],axis=0) + \
            np.sum(vertical[vertical>0],axis=0) + \
            np.sum(diag2[diag2>0],axis=0) + \
            np.sum(diag1[diag1>0],axis=0)

    def initial_temp( node ):
        tmp = np.ones(shape=node.gene.shape,dtype="uint8")
        return float(node.objective_function( tmp ))

    def dependant_initial_t( temperature,node,neighbor ):
        # Increase the temperature to make this neighbor a very likely choice
        # This function may not decrease the temperature

        x = temperature/(abs(node.cost-neighbor.cost)+1)*math.log1p(100/99)
        return temperature/x if 1>x>0 else temperature

    def reduce_t( temperature,repetitions ):
        # common function for temperatur reduction
        return temperature/math.log(repetitions+2,2)

    def sa( start_node, goal_cost, k ):
        node = start_node
        d = start_node.gene.shape[0]**2 # number of repetitions pr temperature value
        temperature = initial_temp( node )

        repetitions = 0
        while temperature>1e-100 and node.cost > goal_cost:
```

```
neighbor = Node( node.random_neighbour(),k ) # choose random neigh
if neighbor.cost < node.cost:
    node = neighbor
else:
    annealing = random.uniform(0,1)
    if annealing < math.pow(math.e,-(neighbor.cost-node.cost)/temperature):
        node = neighbor
repetitions += 1

if repetitions<d:
    # set a dependant initial temperature
    temperature = dependant_initial_t( temperature,node,neighbor )
elif repetitions%d==0:
    temperature = reduce_t( temperature,repetitions )
#end while
return node

n=6 # box size
x = 12 # eggs
k = 2
gene = np.array([1]*x+[0]*(int(n**2)-x)).reshape(n,n)

print "Simulated Annealing - (%dX%d) box of %d eggs width k=%d:" % (n,n,x,k)

print sa(
    Node( gene,k ),
    goal_cost = 0,
    k = k
).gene
```