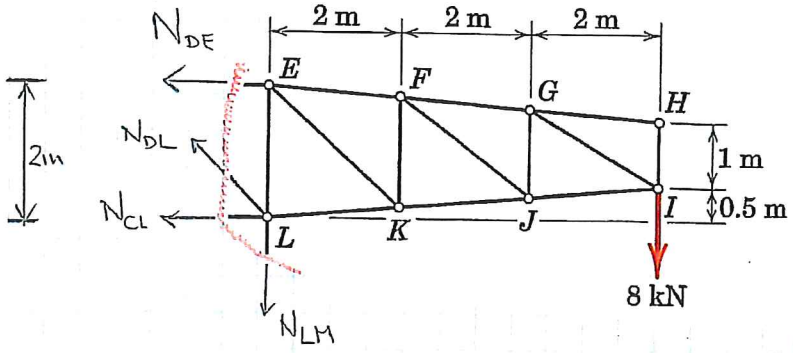
OPPGAVE 1

Definerer et snitt gjennom stavene DE, DL, CL og LM



Momentlikevekt om L

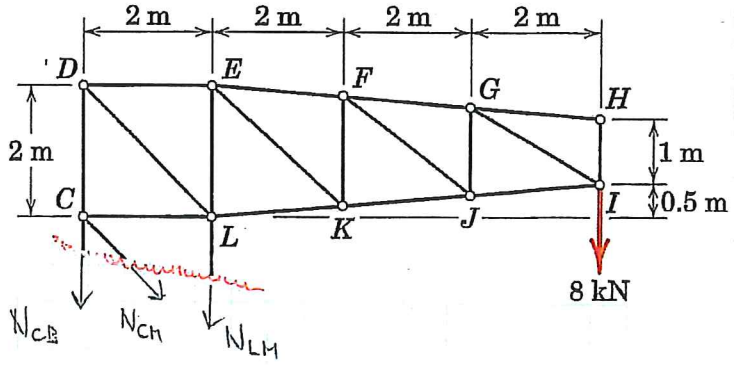
isolerer N_{DE} (de øvrige kreftene angriper gjennom L):

$$\Sigma M_L = 0 \Rightarrow -N_{DE} \cdot 2 + 8 \cdot 6 = 0 \qquad \Rightarrow \quad \underline{N_{DE} = 24\,kN \leftarrow \square \rightarrow}$$

↑ YTRE LAST

Nytt snitt, nå gjennom stavene BC, CM og LM

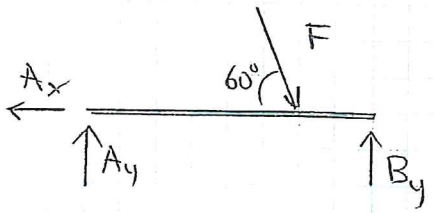
Isolerer N_{LM} med momentlikevekt om C:



$$\Sigma M_C = 0 \Rightarrow N_{LM} \cdot 2 + 8 \cdot 8 = 0 \qquad \Rightarrow \quad \underline{N_{LM} = -32\,kN} \quad \downarrow \square \uparrow$$

OPPGAVE 2 (A)

Lagerreaksjoner:



$\Sigma F_x = 0 \Rightarrow -A_x + F\cos 60° = 0 \Rightarrow \underline{A_x = \frac{1}{2}F}$

$\Sigma M_A = 0 \Rightarrow F \cdot \sin(60°) \cdot 0.75L - B_y \cdot L = 0 \Rightarrow \underline{B_y = \frac{3}{8}\sqrt{3}F = 0{,}650F}$

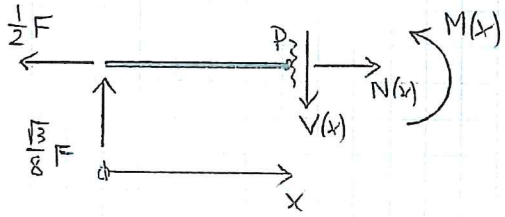
$\Sigma M_B = 0 \Rightarrow A_y \cdot L - F \cdot \sin 60° \cdot 0{,}25L = 0 \Rightarrow \underline{A_y = \frac{1}{8}\sqrt{3}F = 0.217F}$

- Snitt mellom A og punktlast $(0 \le x < 0{,}75L)$:

$\Sigma F_x = 0 \Rightarrow \underline{N_1(x) = \frac{1}{2}F}$

$\Sigma F_y = 0 \Rightarrow \underline{V_1(x) = \frac{\sqrt{3}}{8}F}$

$\Sigma M_P = 0 \Rightarrow \underbrace{\frac{\sqrt{3}}{8}F}_{A_y} \cdot x - M_1(x) = 0 \Rightarrow \underline{M_1(x) = \frac{\sqrt{3}}{8}Fx}$



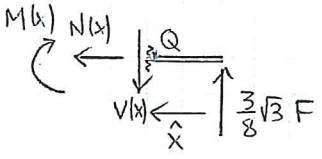
- Snitt mellom B og punktlast $(0 \le \hat{x} < 0.25L)$:

Lar $\hat{x}$ løpe fra B mot venstre

$\Sigma F_x = 0 \Rightarrow \underline{N_2(\hat{x}) = 0}$

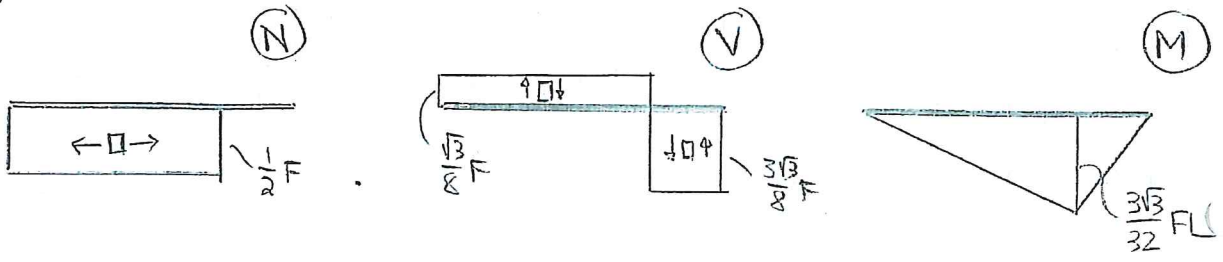
$\Sigma F_y = 0 \Rightarrow \underline{V_2(\hat{x}) = \frac{3}{8}\sqrt{3}F}$

$\Sigma M_Q = 0 \Rightarrow M_2(\hat{x}) - \frac{3}{8}\sqrt{3}F \cdot \hat{x} = 0 \Rightarrow \underline{M_2(\hat{x}) = \frac{3}{8}\sqrt{3}F\hat{x}}$



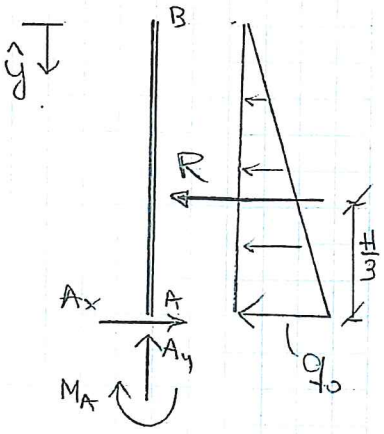
NB: Under punktlast: $M_1(\frac{3}{4}L) = \frac{\sqrt{3}}{8}F \cdot \frac{3}{4}L = \frac{3\sqrt{3}}{32}FL = M_2(\frac{1}{4}L)$

Diagram:



NB: Sprang i (V) under punktlast $= \left(\frac{\sqrt{3}}{8} + \frac{3\sqrt{3}}{8}\right)F = \frac{\sqrt{3}}{2}F = \underbrace{F \cdot \sin 60°}_{\text{VERTIKALKOMPONENT}}$

OPPGAVE 2(B)



Totalresultant av jordtrykk:

$$R = \frac{1}{2} \cdot q_0 \cdot H$$

Funksjon for jordtrykk ($\hat{y}$ løper fra B mot A):

$$q(\hat{y}) = q_0 \frac{\hat{y}}{H}$$

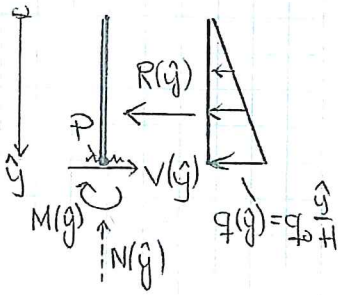
Lagerreaksjoner:

$$\Sigma F_x = 0 \Rightarrow A_x = R = \frac{1}{2} q_0 H$$

$$\Sigma F_y = 0 \Rightarrow A_y = 0$$

$$\Sigma M_A = 0 \Rightarrow -R \cdot \frac{H}{3} + M_A = 0 \Rightarrow M_A = \frac{1}{6} q_0 H \cdot \frac{H}{3} = \frac{1}{6} q_0 H^2$$

Her er det enklest å betrakte et snitt definert ved $\hat{y}$ (løper ovenfra):



$$\Sigma F_x = 0 \Rightarrow V(\hat{y}) - R(\hat{y}) = 0$$

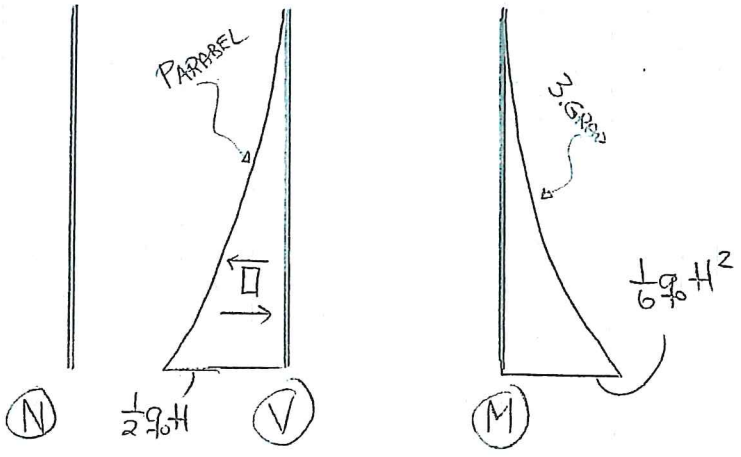
$$\Rightarrow V(\hat{y}) = \frac{1}{2} q_0 \frac{\hat{y}}{H} \cdot \hat{y} = \underline{\frac{1}{2} \frac{q_0 \hat{y}^2}{H}}$$

$$\Sigma F_y = 0 \Rightarrow N(\hat{y}) = 0$$

$$\Sigma M_P = 0 \Rightarrow M(\hat{y}) - R(\hat{y}) \cdot \underbrace{\frac{\hat{y}}{3}}_{\text{ARM}} = 0 \Rightarrow M(\hat{y}) = \underbrace{\frac{1}{2} \frac{q_0 \hat{y}}{H} \hat{y}}_{R} \cdot \frac{\hat{y}}{3}$$

$$\underline{M(\hat{y}) = \frac{1}{6} \frac{q_0 \hat{y}^3}{H}}$$

Diagram:



NB: Innspenning A tilsvarer $\hat{y} = H$

$$V(H) = \frac{1}{2} q_0 H$$

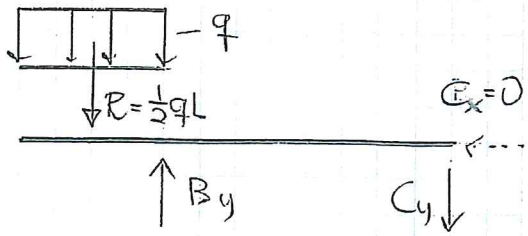
$$M(H) = \frac{1}{6} q_0 H^2$$

Observerer:

$$V(H) = A_x$$

$$M(H) = M_A$$

OPPGAVE 2(C)



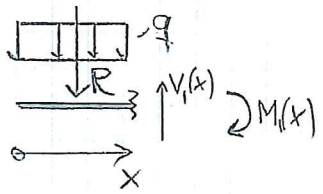
Åpenbart: $C_x = 0$ (og $N \equiv 0$)

Global likevekt:

$$\Sigma M_B = 0 \Rightarrow -R \cdot \frac{L}{4} + C_y \cdot L = 0 \Rightarrow C_y = \frac{1}{8}qL$$

$$\Sigma F_y = 0 \Rightarrow -R + B_y - C_y = 0 \Rightarrow B_y = \frac{5}{8}qL$$

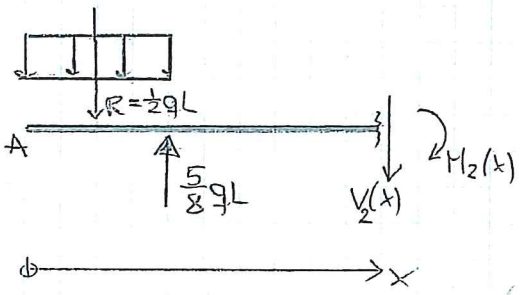
- Snitt mellom A og B:



$$\Sigma F_y = 0 \Rightarrow V_1(x) - R = 0 \qquad V_1(x) = q \cdot x \uparrow\downarrow$$

$$\Sigma M_{snitt} = 0 \Rightarrow M_1(x) - R \cdot \frac{x}{2} = 0 \qquad M_1(x) = \frac{1}{2}qx^2$$

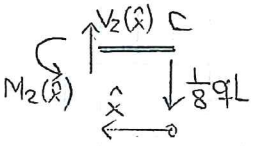
- Snitt mellom B og C:



$$\Sigma F_y = 0 \Rightarrow -\underbrace{R}_{\frac{1}{2}qL} + \underbrace{\frac{5}{8}qL}_{B_y} - V_2(x) = 0 \Rightarrow V_2 = \frac{1}{8}qL \uparrow\downarrow$$

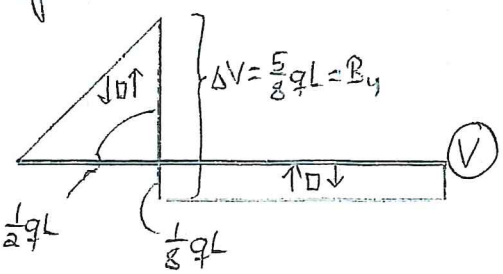
$$\Sigma M_{snitt} = 0 \Rightarrow M_2(x) + \underbrace{\frac{5}{8}qL}_{B_y} \cdot \left(x - \frac{L}{2}\right) - \underbrace{\frac{1}{2}qL}_{R} \cdot \left(x - \frac{L}{4}\right) = 0 \Rightarrow M_2(x) = -\frac{1}{8}qLx + \frac{3}{16}qL^2$$

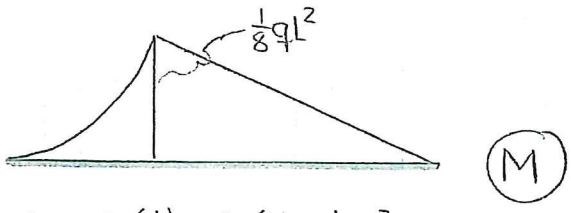
KOMMENTAR: Enklere uttrykk hvis man går inn mot snittet fra høyre, dvs. fra C



NB: $x = \frac{3}{2}L$ i pkt C. Innsatt: $V_2(\frac{3}{2}L) = \frac{1}{8}qL = C_y$ og $M_2(\frac{3}{2}L) = 0$ (R) LEDDLAGET I C

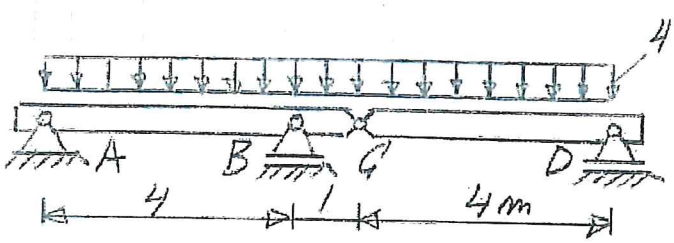
Diagram:





NB: $M_1(\frac{L}{2}) = M_2(\frac{L}{2}) = \frac{1}{8}qL^2 = M_B = M_{max}$

OPPGAVE 3



Skjærkraft $V(x)$ og bøyemoment $M(x)$ Diagrammer.

Likevekt av bjelke ABC $\Rightarrow$

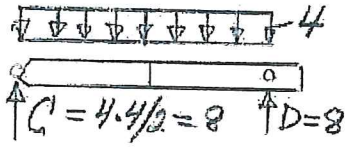
$\Sigma M_A = 0 \Rightarrow B\cdot 4 - C\cdot 5 - 4\cdot 5\cdot \frac{5}{2} = 0$

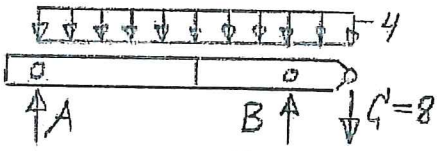
$\Sigma F = 0 \Rightarrow A + B - C - 4\cdot 5 = 0$

$\Rightarrow B = 22{,}5$, $A = 5{,}5$

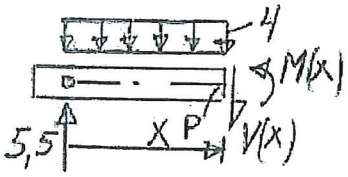
$\Sigma M_B = 0 = \overbrace{4\cdot 5\cdot 1{,}5}^{30} - 8\cdot 1 - A\cdot 4 = 0$

$A = \frac{22}{4} = 5{,}5$ kN





Likevekt av bjelkedel x $(x \le 4m)$



$\Rightarrow \Sigma F = 0 \Rightarrow V(x) = 5{,}5 - 4x$

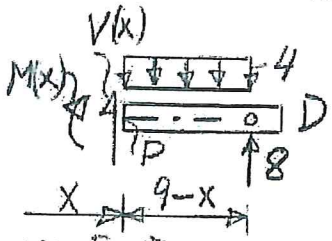
$\Sigma M_P = 0 \Rightarrow M(x) - 5{,}5\cdot x + (4\cdot x)(x/2) = 0$

$\Rightarrow M(x) = 5{,}5x - 2x^2$ [kN]

$V_{max} = 5{,}5$ kN for $x = 0$, $V_{min} = -10{,}5$ for $x = 4$ m

$V = 0$ for $x = 5{,}5/4 = 1{,}375$ m $\Rightarrow M_{max} = 3{,}78$ kNm

$M_{min} = -10$ kNm for $x = 4$ m, $M = 0$ for $x = 2{,}75$ m



Likevekt av bjelkedel $(9-x)$ for $4 \le x \le 9$ m $\Rightarrow$

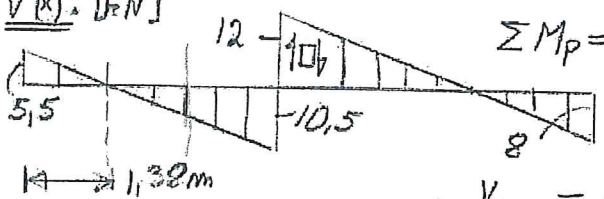
$\Sigma F = 0 \Rightarrow V(x) - 4\cdot(9-x) + 8 = 0$

$\Rightarrow V(x) = 28 - 4x$ [kN]

$\Sigma M_P = 0 \Rightarrow M(x) + [4\cdot(9-x)]\cdot(9-x)/2 - 8\cdot(9-x) = 0$

$\Rightarrow M(x) = -90 + 28x - 2x^2$ [kNm]

$V(x)$: [kN]

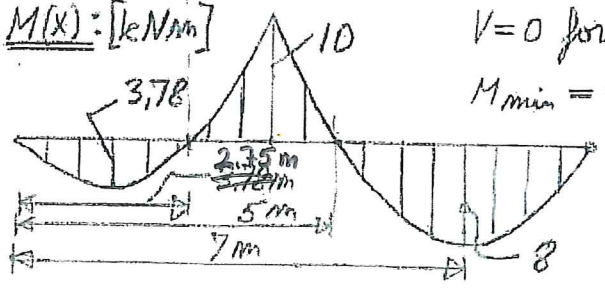


$V_{max} = 12$ kN for $x = 4$ m, $V_{min} = -8$ kN for $x = 9$ m

$V = 0$ for $x = 7$ m $\Rightarrow M_{max} = 8$ kNm

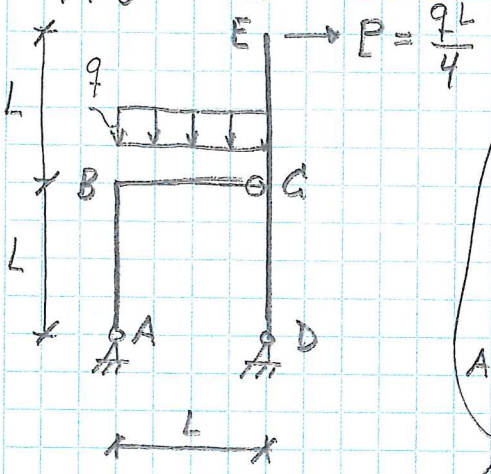
$M_{min} = -10$ kNm for $x = 4$ m, $M = 0$ for $x = 5$ m

$M(x)$: [kNm]



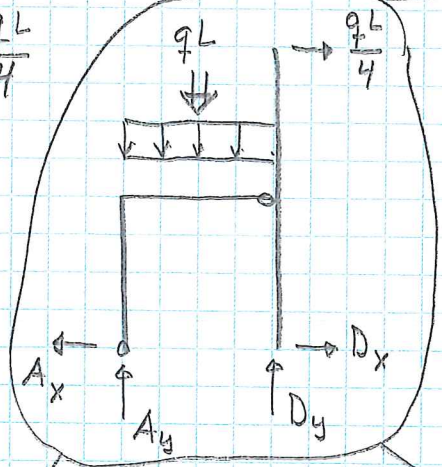
Moment-diagrammet er tegnet på strekksiden.

Oppgave 4



$\rightarrow P = \frac{qL}{4}$

FLD



① $\circlearrowleft \Sigma M_A = 0 \Rightarrow$

$qL\frac{L}{2} + \frac{qL}{4}\cdot 2L - D_y L = 0$

$\Rightarrow \underline{D_y = qL}$

$\uparrow \Sigma K_y = 0 \Rightarrow$

$-qL + A_y + D_y = 0$

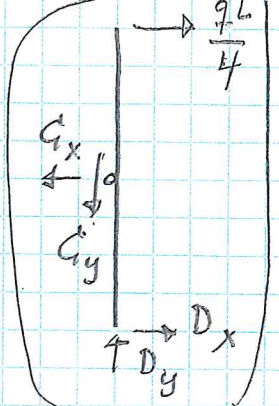
$\Rightarrow \underline{A_y = 0}$

③ $\rightarrow \Sigma K_x = 0 \Rightarrow$

$\frac{qL}{4} - A_x + D_x = 0$

$\Rightarrow \underline{A_x = \frac{qL}{2}}$

FLD



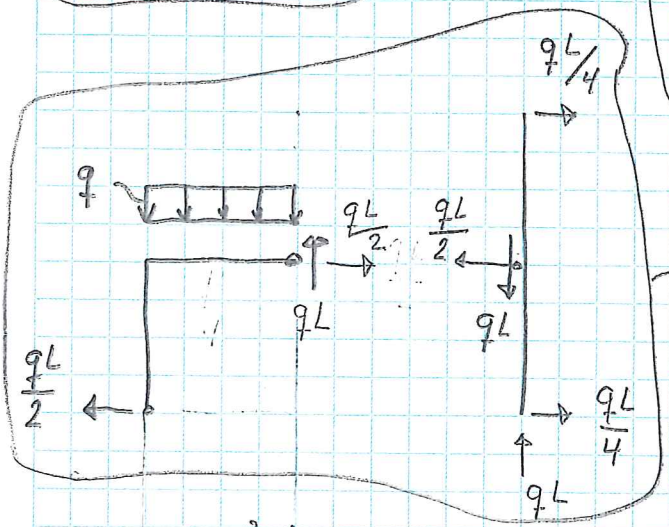
② $\circlearrowleft \Sigma M_C = 0 \Rightarrow$

$\frac{qL}{4}\cdot L - D_x \cdot L = 0$

$\Rightarrow \underline{D_x = \frac{qL}{4}}$

Og: $C_y = D_y = qL$

$C_x = \frac{qL}{4} + D_x = \frac{qL}{2}$



Diagrammene (M, V og N) tegnes direkte opp ifra FLD med påsatte verdier.

Husk: $V = \frac{dM}{ds}$