Task 1

Task 1.a

The signal is given by $V = V_m \cos(\omega t)$ as shown in the figure. Find V_m and ω.

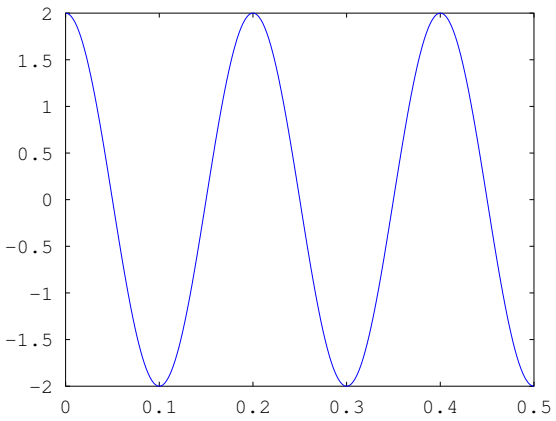

Figure 1: $V = V_m \cos(\omega t)$

Task 1.b

Find the period T and the frequency $f = \frac{1}{T}$. Show that $\omega = 2\pi f$.

Task 1.c

What is the amplitude, and what is the peak-to-peak value?

Task 1.d

The signal is changed to

$$V = V_m \cos(\omega t + \phi)$$

where ϕ is the phase angle. Find the phase angle from the figure

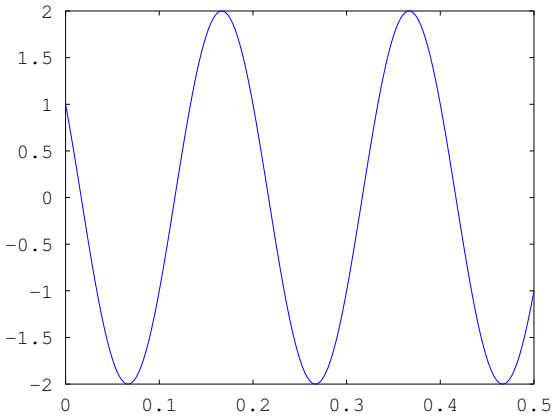

Figure 2: $V_m \cos(\omega t + \phi)$

Task 1 Solution

Task 1.a Solution

By looking at the figure, it can be seen that the amplitude of the function is 2, since the function oscillates between 2 and -2. Therefore $V_m = 2$

By looking at the graph we notice that the cosine function performs 5 cycles in 1 s. This means that ωt goes from 0 to $5 \cdot 2\pi$ when t goes from 0 s to 1 s, hence $\omega = 10\pi$.

Task 1.b Solution

The period is found again from the figure, and is 0.2. The frequency is $f = \frac{1}{T} = \frac{1}{0.2} = 5$. We then find that $2\pi f = 10\pi$ which is the same as ω from the previous task.

Task 1.c Solution

The amplitude is the same as V_m and is therefore 2. Peak-to-peak value is the difference between the top and bottom value. This is also given as two times the amplitude, in this case 4.

Task 1.d Solution

By looking at the figure, it can be seen that the function crosses the y-axis at 1 when x is 0. This means that $2\cos(\phi) = 1$ which leads to $\phi = \arccos(\frac{1}{2}) = \frac{\pi}{3} \approx 1.0472$

Task 2

The AC current is given by the phasor

$$I = I_0 \exp j\omega t$$

where $I_0 = 1\,\text{mA}$ and the period is $T = 0.01\,\text{s}$. It is given that $\omega T = 2\pi$.

Task 2.a

What is the real current corresponding to the phasor I?

Task 2.b

The current runs through an impedance

$$Z = \frac{1}{j\omega C}, \quad \text{where } C = 1\,\mu\text{F}$$

which is depicted in Figure 3. What is the voltage phasor V?

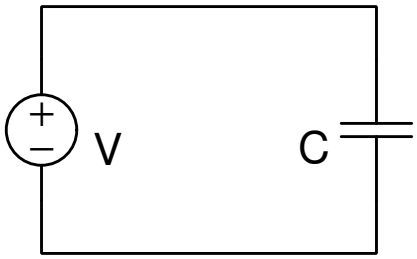

Figure 3: Capacitor circuit

Task 2.c

What is the real voltage?

Task 2.d

A resistor $R = 2\,\mathrm{k\Omega}$ is inserted in series with the capacitor, which is depicted in Figure 4. The current is still $I = I_0 e^{j\omega t}$. What is the impedance of the two circuit elements combined?

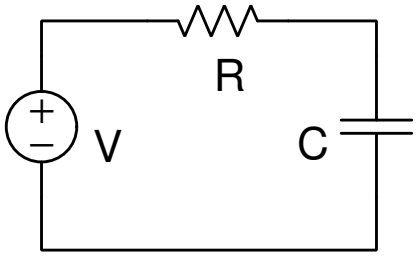

Figure 4: RC circuit

Task 2.e

What is the resulting voltage phasor?

Task 2 Solution

Task 2.a Solution

The real current is defined as $Re[I]$. By using Euler's formula $e^{jax} = \cos(ax) + j\sin(ax)$ we see that I can be written as $I = I_0 e^{j\omega t} = I_0(\cos(\omega t) + j\sin(\omega t))$. The real value is therefore $Re[I] = I_0\cos(\omega t) = \cos(200\pi t)\mathrm{mA}$

Task 2.b Solution

We have that $I_0 e^{j\omega t}$ and $Z = \frac{1}{j\omega C}$ which we use to calculate the voltage

$$V = IZ = I_0 \exp(j\omega t)\frac{1}{j\omega C} = \frac{1\,\mathrm{mA}}{1\,\mu\mathrm{F}\cdot\omega}\frac{\exp(j\omega t)}{j} = \frac{5}{\pi}\exp(j200\pi t - j\frac{\pi}{2})\mathrm{V}$$

Task 2.c Solution

By Eulers formula we find the real part of the voltage phasor

$$V_{Re} = Re[V] = \frac{5}{\pi}\cos(200\pi t - \frac{\pi}{2})\text{V}$$

Task 2.d Solution

$$Z_{TOT} = Z_R + Z_C = R + \frac{1}{j\omega C} = R - \frac{j}{\omega C} = 2\,\text{k}\Omega - j\frac{1}{\pi\, 200\,\text{s}^{-1} \times 1\,\mu\text{F}} = \left(2000 - j\frac{5000}{\pi}\right)\Omega$$

Task 2.e Solution

$$\begin{aligned} V &= ZI = I_0 \exp(j\omega t)(R - j\frac{1}{\omega C}) \\ Z &= R - j\frac{1}{\omega C} = R - \frac{1}{\omega C}\exp(\frac{j\pi}{2}) \\ V &= R\exp(j\omega t) - \frac{1}{\omega C}\exp(j(\omega t + \frac{\pi}{2})) \\ &= 2\exp(j200\pi t) - \frac{5}{\pi}\exp(j200\pi t + j\frac{\pi}{2})\text{V} \end{aligned}$$

Task 3

Given the parts in Figure 5 (PINX, 5V and GND are pins on a microcontroller) and the code in Code 1, make the LED blink.

Code 1: Blink code for microcontroller

```
void setup(){
  pinMode(11, OUTPUT);
}

void loop(){
  digitalWrite(11, HIGH);
  delay(1000);
  digitalWrite(11, LOW);
  delay(1000);
}
```

Task 3 Solution

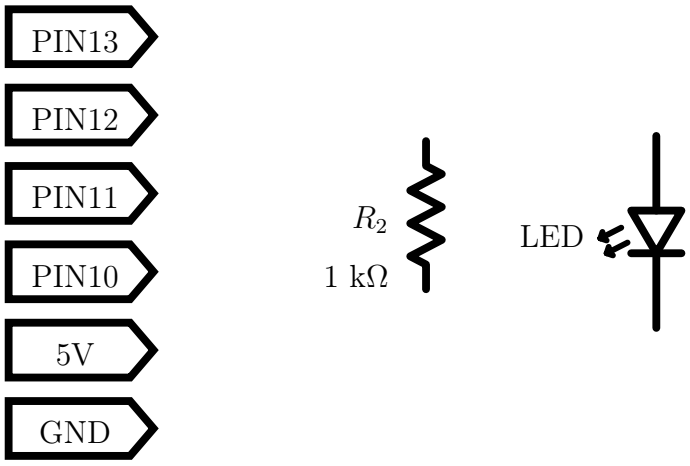

Figure 5: Components for circuit

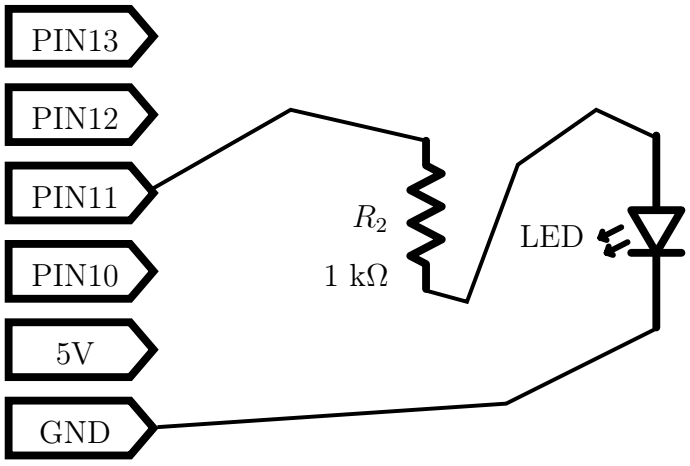

Figure 6: Solution for task 3