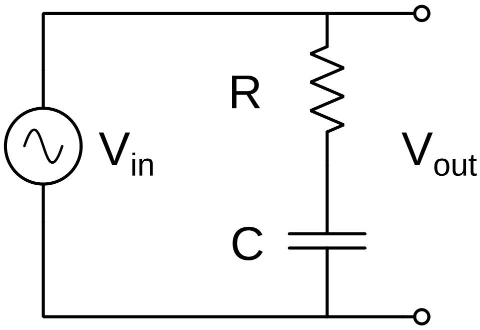
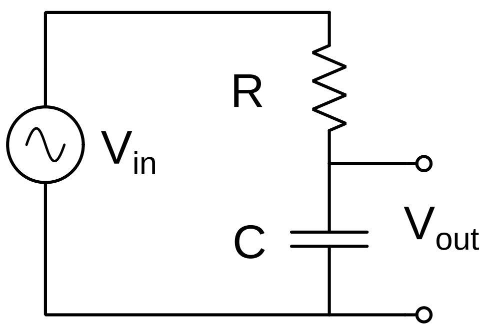
Task 1



(a) RC circuit 1



(b) RC circuit 2

Figure 1

Task 1.a

Consider the circuit in Figure 1(a). Find the voltage V_{out} when then input voltage V_{in} is given by $V_{\text{in}} = V_m e^{j\omega t}$.

Task 1.b

Consider the circuit in Figure 1(b). Compute the complex impedance of the circuit elements R and C individually, and the combined impedance of both circuit elements.

Task 1.c

Compute the voltage phasor over the capacitor V_{out}. V_{in} is still given by $V_{\text{in}} = V_m e^{j\omega t}$.

Task 1.d

The impedance value of the RC circuit is frequency dependent. The specific values of the circuit elements are given, $C = 1\,\mu\text{F}$ and $R = 1\,\text{k}\Omega$. Compute the values that fill in the table.

Frequency ω	Impedance ($Z_{combined}$)	Voltage (V_{out})
$10\,\text{rad}\,\text{s}^{-1}$		
$100\,\text{rad}\,\text{s}^{-1}$		
$1000\,\text{rad}\,\text{s}^{-1}$		
$10\,000\,\text{rad}\,\text{s}^{-1}$		
$100\,000\,\text{rad}\,\text{s}^{-1}$		

Table 1

Describe briefly in words what the circuit does to the voltage signal V_{in}, i.e. the transfer function from V_{out} to V_{in}.

Task 1 Solution

Task 1.a Solution

There is no voltage drop between V_{IN} and V_{OUT}, which means that $V_{OUT} = V_{IN} = \underline{V_m \exp j\omega t}$

Task 1.b Solution

First we find the impedance of the circuit elements R and C individually

$$\underline{Z_R = R}, \qquad \underline{Z_C = \frac{1}{j\omega C}}.$$

The equivalent impedance of the series connection of R and C is

$$Z_{combined} = Z_R + Z_C = R + \frac{1}{j\omega C}.$$

Task 1.c Solution

Lets first find the current running through the circuit which is

$$I = \frac{V_{in}}{Z_{combined}}.$$

We then use Ohms law to find the voltage over the capacitor, i.e. V_{out}

$$V_{out} = Z_C I = V_m e^{j\omega t} \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = V_m e^{j\omega t} \frac{1}{j\omega CR + 1}.$$

$$\begin{aligned} V_{out} &= V_m e^{j\omega t} \frac{1}{j\omega CR + 1} \cdot \frac{j\omega CR - 1}{j\omega CR - 1} \\ &= V_m e^{j\omega t} \frac{1 - j\omega CR}{\omega^2 C^2 R^2 + 1} \end{aligned}$$

The term $1 - j\omega CR$ is a complex number with magnitude $\sqrt{1 + \omega^2 C^2 R^2}$ and phase $\arctan(-\omega CR)$, thus

$$V_{out} = \frac{V_m}{\sqrt{1 + \omega^2 C^2 R^2}} e^{j\omega t} e^{j \arctan(-\omega CR)}.$$

Task 1.d Solution

We insert the values we know into the expression

$$V_{out} = \frac{V_m}{\sqrt{1 + \omega^2 10^{-6}}} e^{j\omega t} e^{j \arctan(-\omega 10^{-3})}.$$

and we insert the interesting part containing amplitude and phase information into the table. By looking at the values in the table we see that the amplitude is decreasing with increasing frequency. We also see that the phase is shifting from $\arctan(-10^{-2}) \approx -0.06°$ to $\arctan(-100) \approx -89.43°$. This is typical characteristics of a low pass filter.

Frequency ω	Impedance ($Z_{combined}$)	Voltage (V_{out})
$10\,\text{rad}\,\text{s}^{-1}$	$1\,\text{k}\Omega - j10^5\Omega$	$\frac{1}{\sqrt{1+10^{-4}}}e^{j\arctan(-10^{-2})}$
$100\,\text{rad}\,\text{s}^{-1}$	$1\,\text{k}\Omega - j10^4\Omega$	$\frac{1}{\sqrt{1+10^{-2}}}e^{j\arctan(-10^{-1})}$
$1000\,\text{rad}\,\text{s}^{-1}$	$1\,\text{k}\Omega - j10^3\Omega$	$\frac{1}{\sqrt{1+1}}e^{j\arctan(-1)}$
$10\,000\,\text{rad}\,\text{s}^{-1}$	$1\,\text{k}\Omega - j10^2\Omega$	$\frac{1}{\sqrt{1+100}}e^{j\arctan(-10)}$
$100\,000\,\text{rad}\,\text{s}^{-1}$	$1\,\text{k}\Omega - j10\Omega$	$\frac{1}{\sqrt{1+10000}}e^{j\arctan(-100)}$

Table 2

Task 2

Task 2.a

Given the circuit in Figure 3, and the signal in Figure 6, draw the path of the current at the following time instants:

1. $t = 1$
2. $t = 2.5$
3. $t = 3.25$

Task 2.b

Draw the output signal of V_{OUT} for t in the interval $[0, 5]$.

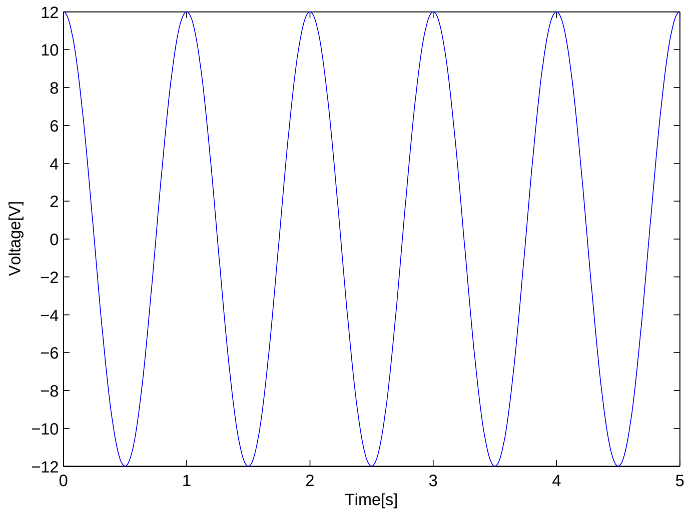


Figure 2: Input signal V_{IN}

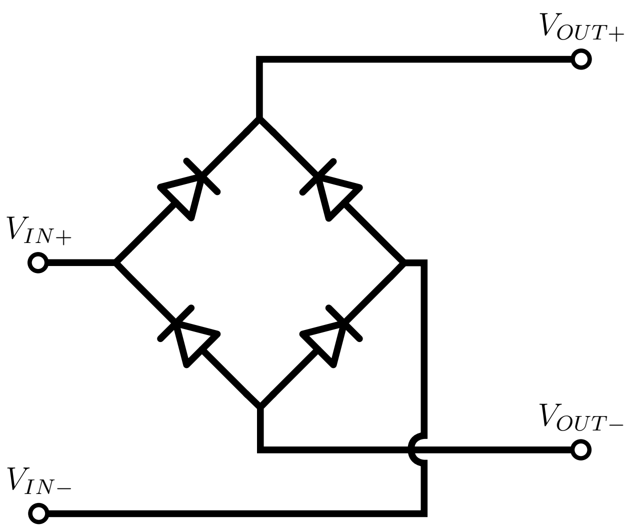


Figure 3: Diode bridge

Task 2 Solution

Task 2.a Solution

1. Depicted in Figure 4
2. Depicted in Figure 5
3. There is no current running through the circuit, because there is no voltage.

Task 2.b Solution

Depicted in Figure 6

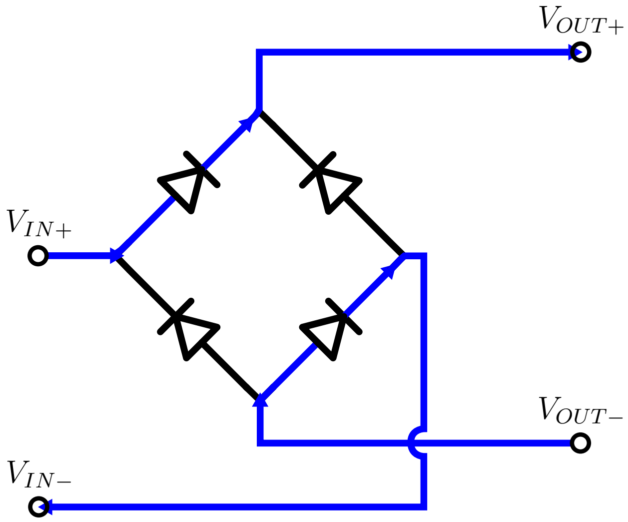


Figure 4: Solution to task 2.a.1

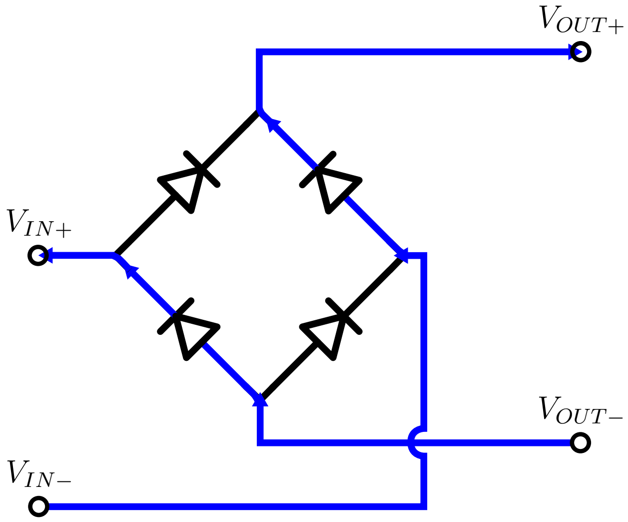


Figure 5: Solution to task 2.a.2

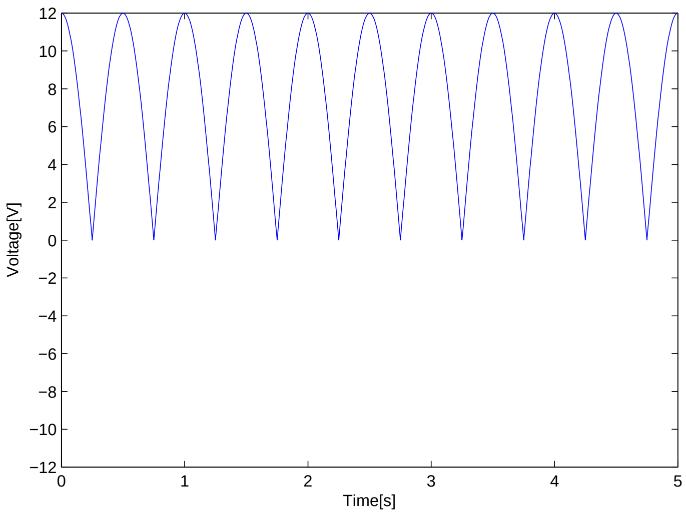


Figure 6: Solution to task 2.b

Task 3

A system is implemented to keep a constant level of liquid within a storage tank. Figure 8 shows a diagram of the system. A valve controls the input of liquid to the tank, and a regulator controls the opening of the valve. The control signal from the regulator is between 0 V and 10 V, where 0 V is completely shut, and 10 V is fully open.

An emergency stop is to be installed in order to prevent overspilling of liquid in the tank. The emergency stop is supposed to stop the current going from the regulator to the valve filling the storage tank.

The emergency stop button is implemented so that when the button is pressed, there is no connection over the button, and the voltage will drop from 5 V to 0 V on the input to both the switch circuit and the warning system.

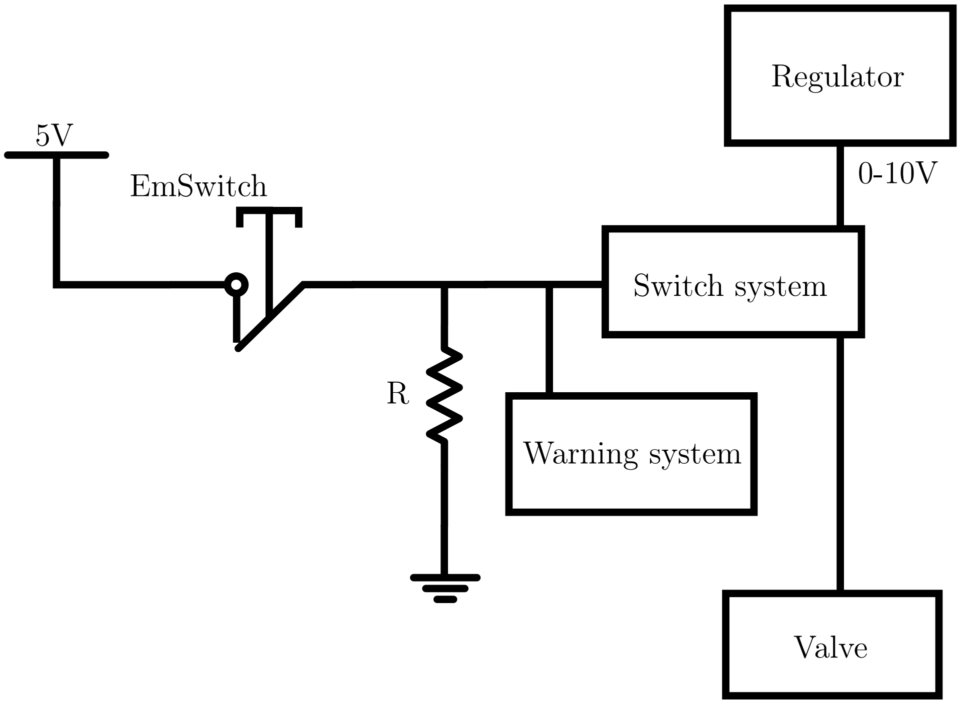


Figure 7: Diagram of liquid level system

Task 3.a

Create the switch system by drawing a transistor switch circuit that breaks the current flowing from the regulator to the valve when the emergency stop button is pressed.

Task 3.b

Create the warning system so that a lamp is lit when the emergency stop button is pressed.

Hint: Use a transistor and a resistor to invert the emergency stop signal to light up the emergency stop lamp.

Task 3 Solution

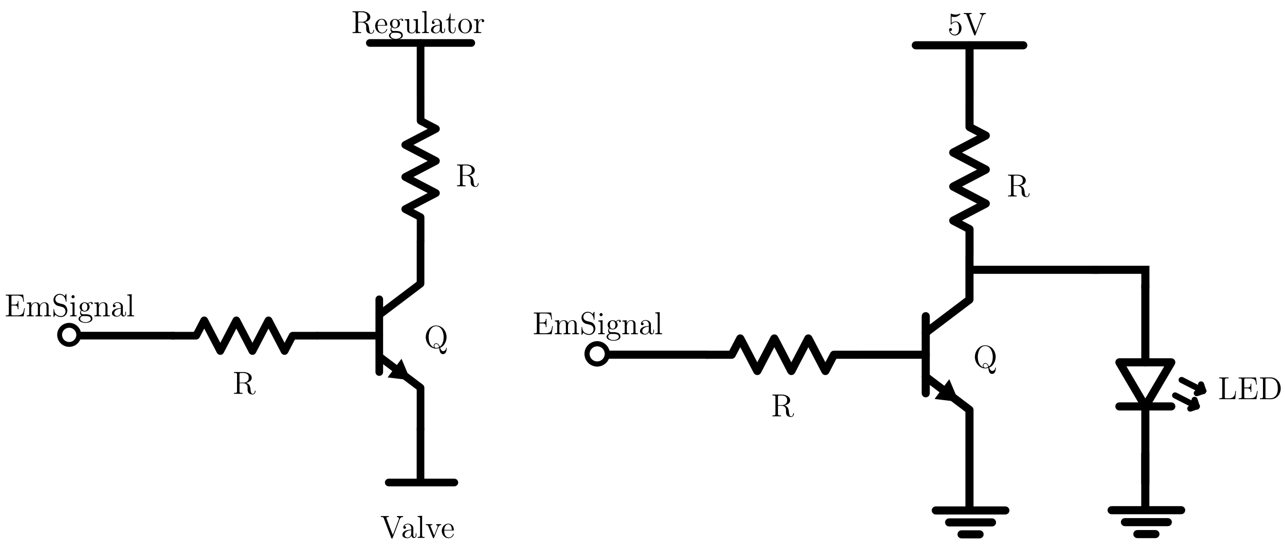
There are many circuits that can produce the desired effects, but here are some proposed circuits. Values on the components are not important.

Task 3.a Solution

Depicted in Figure 8(a).

Task 3.b Solution

Depicted in Figure 8(b).



(a) Proposed solution for task 3.a

(b) Proposed solution for task 3.b

Figure 8: Proposed solution for task 3