Task 1

Task 1.a

4 bytes are given by the hex digits `3A F0 00 00`.

What is the corresponding digits for the 4 bytes?

Task 1.b

A variable is stored in 4 bytes with 32 bits in total. The bits are numbered from 0 to 31 and are $b_0, b_1, \ldots, b_{31}$

The varable is declared as a float with numerical value

$$a = (-1)^{b_{31}}(1 + \sum_{i=1}^{23} b_{23-i}2^{-i}) \cdot 2^{exponent-127}$$

where *exponent* is an unsigned integer in the range 0-255 stored in the 8 bits $b_{23}, b_{24}, \ldots, b_{30}$

What is a when the 4 bytes are given by the hex numbers `3A F0 00 00`?

Task 1 Solution

Task 1.a

The corresponding digits are 00111010111100000000000000000000.

Task 1.b

By using the formula: $a = 0.0018310547$.

Task 2

Task 2.a

What is the function of the circuit? Explain how it works.

Task 2 Solution

Task 2.a Solution

This is a 4 bit data register, where bits b_0, b_1, b_2, b_3 is stored in D-latch 1, 2, 3 og 4.

The D-latch functions so that Q_x will not change if Load is low. If load is high, Q_x will follow D_x. When Read is high, you can read the values of the bits in R_x.

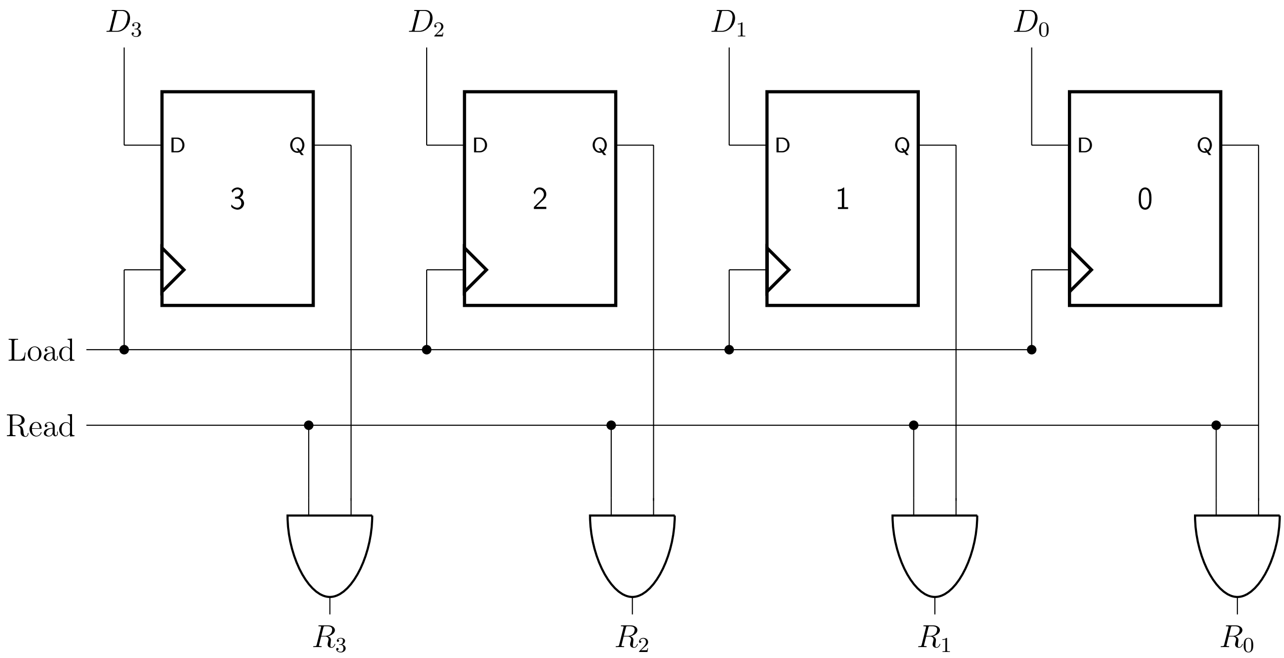

Figure 1: Logic circuit

Task 3

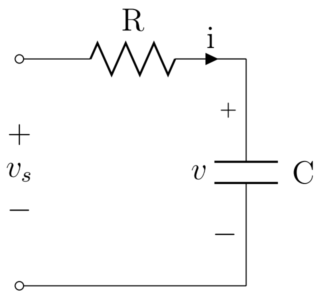

Figure 2: RC circuit

Kirchoff's Voltage Law of Figure 2 gives the differential equation

$$\tau \frac{dv}{dt} = -v + v_s$$

where $\tau = RC$.

Task 3.a

Show that

$$v(t) = V_0 e^{-\frac{t}{\tau}}$$

is a solution to the differential equation when $v_s = 0$ and $v(t = 0) = V_0$.

Make a sketch of $v(t)$, and calculate $v(t)$ for $t = \tau$, $t = 2\tau$ and $t = 5\tau$.

Task 3.b

Show that the tangent to $v(t)$ at $t = 0$ intersects the zero line at $t = \tau$.

Task 3.c

Show that

$$v(t) = V_s(1 - e^{-\frac{t}{\tau}})$$

is a solution when $v_s = V_s$ and $v(t = 0) = 0$.

Make a sketch of $v(t)$.

Task 3.d

Given $C = 1\,\mu\text{F}$ and we want $\tau = 0.1\,\text{s}$. Find R.

If $v_s = 5\,\text{V}$, what is $v(t = \tau)$?

Task 3 Solution

Task 3.a Solution

When $v_s = 0$ we have the equation:

$$\frac{dv}{dt} = -\frac{1}{\tau}v$$

By using the exponential decay equation we get:

$$v(t) = v(0)e^{-\frac{1}{\tau}} = V_0 e^{-\frac{1}{\tau}t}$$

Calculating $t = \tau$:

$$v(t = \tau) = V_0 e^{-1} = \frac{V_0}{e}$$

Calculating $t = 2\tau$:

$$v(t = \tau) = V_0 e^{-2} = \frac{V_0}{e^2}$$

Calculating $t = 5\tau$:

$$v(t = \tau) = V_0 e^{-5} = \frac{V_0}{e^5}$$

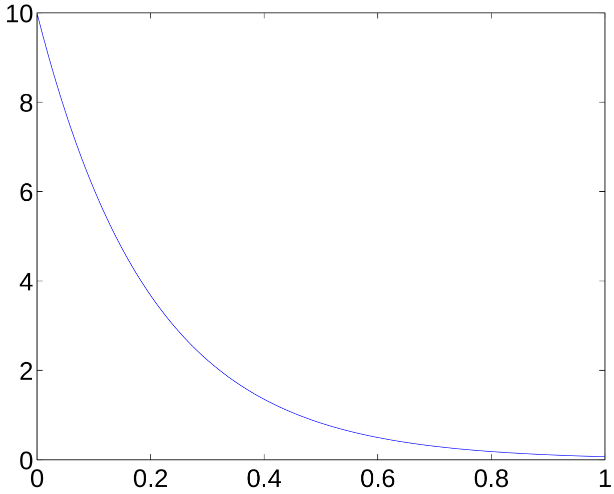

Figure 3: Plot of $v(t)$ with $V_0 = 10\,\mathrm{V}$ and $\tau = \frac{1}{5}$

Task 3.b Solution

The tangent of $v(t = 0)$ is given as:

$$y - V_0 = \frac{dv}{dt}(0) \cdot (t - 0)$$
$$y - V_0 = -V_0 \frac{1}{\tau} t$$
$$y = -V_0 \frac{1}{\tau} t + V_0$$

The zero line i s when $y = 0$:

$$0 = -V_0 \frac{1}{\tau} t + V_0$$
$$V_0 \frac{1}{\tau} t = V_0$$
$$t = \tau$$

Task 3.c Solution

$$\tau \frac{dv}{dt} = -v + v_s$$
$$\Rightarrow \frac{dv}{dt} = -\frac{1}{\tau} v + \frac{1}{\tau} v_s$$

Solving the differential equation we get

$$v(t) = V_s(1 - e^{-\frac{t}{\tau}})$$

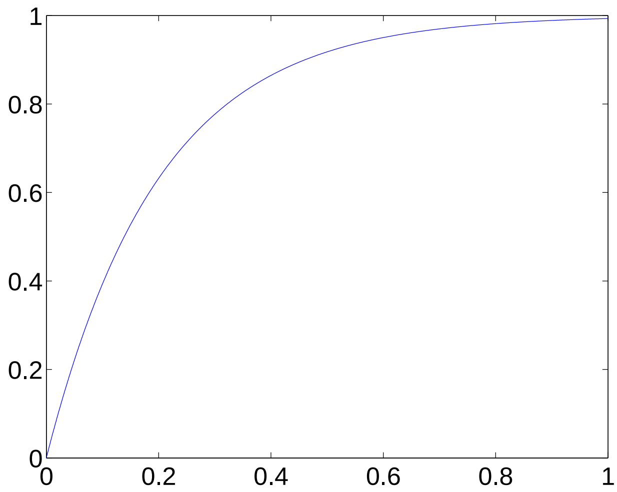

Figure 4: Plot of $v(t)$ with $V_s = 10\,\mathrm{V}$ and $\tau = \frac{1}{5}$

Task 3.d Solution

$\tau = RC \Rightarrow R = \frac{\tau}{C} = \frac{0.1}{10^{-1}} = \underline{\underline{100\,\mathrm{k}\Omega}}$

$$v(t) = V_s(1 - e^{-\frac{t}{\tau}})$$
$$v(t) = 5(1 - e^{-\frac{\tau}{\tau}})$$
$$\underline{\underline{v(t) = 5(1 - e)}}$$

Task 4

Task 4.a

Show by using a truth table that:

$$A + 1 = 1$$
$$A + 0 = A$$
$$A \cdot 1 = A$$
$$A \cdot 0 = 0$$

Task 4 Solution

Task 4.a

It can be seen in Table 1 that column 2 and 3 are equal.

It can be seen in Table 2 that column 1 and 3 are equal.

It can be seen in Table 3 that column 1 and 3 are equal.

It can be seen in Table 4 that column 2 and 3 are equal.

A	1	A + 1
0	1	1
1	1	1

Table 1: Truth table for $A + 1 = 1$

A	0	A + 0
0	0	0
1	0	1

Table 2: Truth table for $A + 0 = A$

Task 5

The pins 0 to 7 on the Arduino are mapped to a 1 byte register called `PORTD`. If bit $b_3 = 0$ in `PORTD`, then pin 3 will be 0 V, while if $b_3 = 1$, then pin 3 will be 5 V.

Task 5.a

Lets assume that the value of `PORTD` is $A7_{16}$. What are the voltages on pins 0-7?

Task 5.b

By using `PORTD = PORTD | x`, you can change a pin from 0 V to 5 V. For instance: `x` $= 8_{16}$ will change pin 4.

Which values of `x` will change pins 7 and 5 if `PORTD` $= A7_{16}$?

Task 5.c

By using `PORTD = PORTD & !x`, you can change a pin from 5 V to 0 V. For instance: `x` $= 4_{16}$ will change pin 3.

Which values of `x` will change pins 8, 6 and 2 if `PORTD` $= A7_{16}$?

Task 5 Solution

Task 5.a

Pins 0, 1, 2, 5 and 7 are 5 V, while 3, 4 and 6 are 0 V.

Task 5.b

In order to change pin 7 we must have `x` $=$ `1 << 7` $= 40_{16}$.

In order to change pin 5 we must have `x` $=$ `1 << 5` $= 10_{16}$.

A	1	$A \cdot 1$
0	1	0
1	1	1

Table 3: Truth table for $A \cdot 1 = A$

A	0	$A \cdot 0$
0	0	0
1	0	0

Table 4: Truth table for $A \cdot 0 = 0$

Task 5.c

In order to change pin 8 we must have `x` = `1 << 8` $= 80_{16}$.

In order to change pin 6 we must have `x` = `1 << 6` $= 20_{16}$.

In order to change pin 2 we must have `x` = `1 << 2` $= 4_{16}$.