Task 1

A capasitor has the characteristic

$$I = C\frac{dV}{dt}$$

The current is defined as

$$I = \frac{dq}{dt}$$

where q is the elctrical charge through the conductor. Therefore, the characteristic can also be written

$$q = CV$$

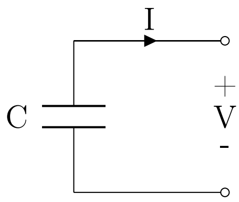

Figure 1: Capacitor circuit

A piezoelectric sensor outputs a charge q prprtional to the force F on the sensor:

$$q = KF$$

The circuit equivalent is given in Figure 2

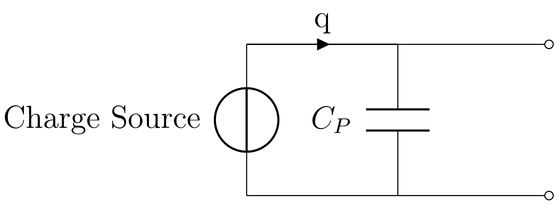

Figure 2: Piezo equivalent circuit

which has the equivalent Thevenin representation shown in Figure 3

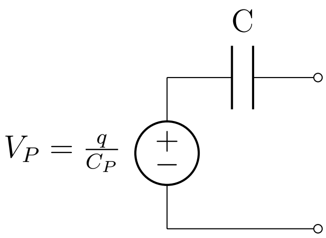

Figure 3: Piezo equivalent circuit

A charge amplfier is depicted in Figure 4 where C_C is the capacitance of the cable.

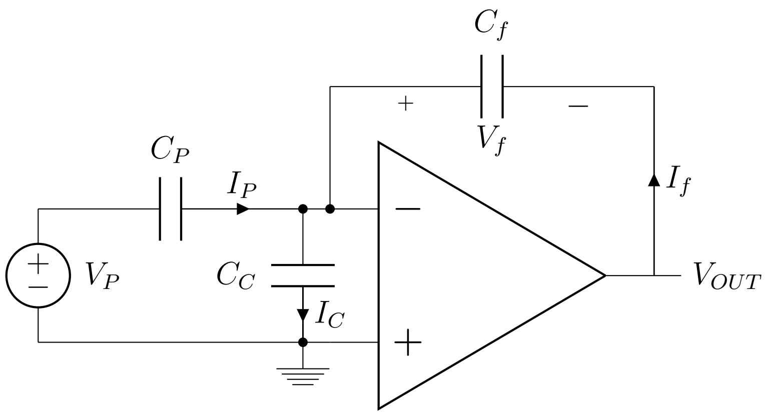

Figure 4: Charge amplifier circuit

Task 1.a

Show that $V_f = -V_{OUT}$.

Task 1.b

Show that $I_c = 0$.

Task 1.c

Show that $I_f = -I_P$.

Task 1.d

Show that $V_{OUT} = -\frac{K}{C_P}F$.

Task 1 Solution

Task 1.a

Since the positive input of the amplifier is connected to ground, the negative input will be 0 V. This means that C_f is connected to V_{OUT} in one end and 0 V in the other. This leads to $\underline{\underline{V_f = -V_{OUT}}}$.

Task 1.b

Since there is no current going between the positive and negative input of the amplifier, $\underline{\underline{I_C \text{ will be } 0}}$.

Task 1.c

Since there is no current going between the positive and negative input of the amplifier, $I_P + I_f = 0 \Rightarrow \underline{\underline{I_f = -I_P}}$.

Task 1.d

$q = C_P V_P$ and $q = KF$ means $V_P = \frac{K}{C_P}F$. This means that

$$\begin{aligned} I_P &= -I_f \\ C_P \frac{dV_P}{dt} &= -C_f \frac{dV_{OUT}}{dt} \\ -\frac{C_P}{C_f} V_P &= V_{OUT} \\ V_{OUT} &= -\frac{C_P}{C_f}\frac{KF}{C_P} = \underline{\underline{-\frac{K}{C_f}F}} \end{aligned}$$

Task 2

An accelerometer has a proof mass m with position x_0. he mass is connected with a spring and a damper to the base, which has position x_i. The accelerometer output is the relative position

$$x_r = x_0 - x_i$$

of the proof mass. The accelerometer is used to measure the acceleration

$$a_i = \frac{d^2}{dt^2}x_i$$

Suppose that $m = 0.001\,\mathrm{kg}$ and $k = 40\,000\,\mathrm{N\,m^{-1}}$

Task 2.a

What is the natural frequency of the spring-damper system?

Task 2.b

Suppose that frequencies $x_i(t) = X_0 cos(2\pi f t)$. For what range of f will the accelerometer give an accurate measurement of a_i?

Task 2 Solution

Task 2.a Solution

The natrual frequency is given as

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{40000}{0.001}} = \underline{\underline{40\,000\,000\,\mathrm{s\,m^{-1}}}}$$

Task 2.b Solution

As long as the frequency f in x_i is such that $f < \omega_n = 40\,000\,000\,\mathrm{s\,m^{-1}}$, the accelerometer will give an accurate measurement of a_i.

Task 3

B_0	B_1	B_2
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

(a) Binary code

G_0	G_1	G_2
0	0	0
0	0	1
0	1	1
0	1	0
1	1	0
1	1	1
1	0	1
1	0	0

(b) Gray code

Figure 5: Binary code and gray code

Digital optical encoders (textbook p. 383) use gray code so that only one bit is changed at the time when counting from 0 to $2^n - 1$, where n is the number of bits.

Task 3.a

Find the logic gates of the gray code to convert G_0, G_1 and G_2 to B_0, B_1 and B_2 when $n = 3$. of the binary code.

Task 3 Solution

Task 3.a

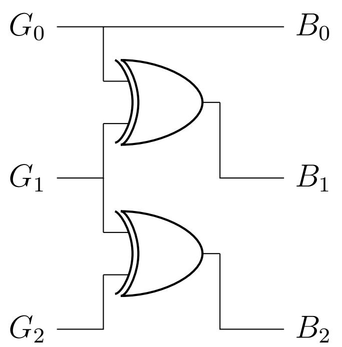

Figure 6: Logic gates to convert from G_0, G_1 and G_2 to B_0, B_1 and B_2

The logic gates which converts G_0, G_1 and G_2 to B_0, B_1 and B_2 is shown in Figure 6.